### MHD Effects on Blood Flow in a Stenosis

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#### Abstract

The fundamental problem of blood flow in a channel with stenosis under the influence of a steady uniform magnetic field is studied. The mathematical model used for the formulation of the problem is consistent with the principles of Magnetohydrodynamics (MHD). Blood is considered as a homogeneous Newtonian fluid and is treated as an electrically conducting magnetic fluid. The finite volume (FV) discretization scheme in curvilinear coordinates is used for the discretization of the system of equations governing the MHD blood flow. For the numerical solution of the problem, which is described by a coupled, nonlinear system of PDEs, with appropriate boundary conditions, the SIMPLE method is adopted. Results concerning the velocity, pressure, and skin friction indicate that the presence of the magnetic field influences considerably the flow field.

**AMS Subject Classifications:** 76W05, 76Z99. **Keywords:** Magnetic fluid, MHD, channel flow, stenosis, blood flow.

### **1** Introduction

According to recent estimates, coronary artery disease is responsible for one of six deaths in the US and the developed countries, causing about one million heart attacks

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each year only in the US. Among these, the rupture of coronary vulnerable plaques (VPs) followed by luminal blockage is recognized as a major cause of sudden heart attacks. In these pathological conditions coronary arteries can be occluded by fatty deposits forming the vulnerable plaque (VP) that may trigger rapid thrombosis upon rupture and cause a heart attack [10]. The main objective of this study is to investigate means of reducing severe hemodynamic effects that could lead to VP rupture by developing a mathematical model based on principles of biomagnetic fluid dynamics (BDF). Since blood contains iron, one important question is whether a magnetic field could influence the flow at the stenotic region.

Recently, an extended biofluid dynamics (BFD) mathematical model, which includes the initial BFD model of Haik et al, was developed by Tzirtzilakis [14]. According to this BFD model, a biofluid is considered as a Newtonian, homogeneous, incompressible, and electrically conducting fluid and the flow is considered as laminar. One of the most characteristic biofluids, which exhibits electrical conductivity, is blood [14]. According to the BFD model of [14] the biofluid flow under the influence of an applied magnetic field is consistent with the principles of Magnetohydrodynamics (MHD), [1,2, 12] and Ferrohydrodynamics (FHD) [11].

The fundamental problems of biomagnetic fluid flow in a channel with stenosis and in a lid-driven cavity have been studied in [15] and [16], respectively. The mathematical model used for the formulation of these problems is the one developed in [14]. The results concerning the velocity field, for both problems, showed that the presence of the magnetic field influenced considerably the flow field. The principles of MHD cannot be ignored when a uniform steady magnetic field is applied, either globally or locally, and play a very important role in the formation of the flow field. Overall, the developed mathematical model has been proved useful for understanding the influence of the magnetic field in the study of MHD physical problems [14–16].

In the present study, the MHD flow in a channel with symmetric stenosis is numerically investigated. The flow is assumed to be two dimensional (2D), laminar, incompressible and the magnetic field is uniformly applied globally at the flow field. The two impermeable plates of the channel and the biofluid entering the channel are kept at constant temperatures. The two plates of the channel form a 50% stenosis at the center of the domain. As far as the mathematical model is concerned, the mathematical model described in [14, 16] is taken into account and is valid for the flow like the one in large blood vessels, where the blood can be considered as a homogeneous and Newtonian fluid [9, 14].

The geometry of the stenotic 2D channel requires the development of a generalized nonorthogonal curvilinear coordinate approach. This approach is a local methodology that allows the description of complex geometries such the one used in this study [13]. The biofluid is blood and is considered as a homogeneous, Newtonian and electrically conducting fluid. The physical problem is described by a coupled, nonlinear system of PDEs which is discretized using the finite volume technique and numerically solved applying the semi-implicit method for pressure linked equations (SIMPLE).



Figure 2.1: Schematic representation of the geometry, physical problem configuration and locations where the velocity is presented in the results.

#### 2 Mathematical Formulation

The viscous, steady, 2-dimensional, incompressible, laminar, biomagnetic fluid (blood) flow is considered taking place between two impermeable plates forming a constriction. The length of the plates is  $\overline{L}$  and the distance between them at the entrance is  $\overline{D}$ , as depicted in Figure 2.1. The positions of the upper and the lower plate, are formulated mathematically by the use of a function with respect to the  $\overline{x}$  direction and more details can be found elsewhere [3].

The flow is subject to a uniform magnetic field with direction perpendicular to the  $\bar{x}$  direction. A schematic representation of the geometry and the flow field is given in Figure 2.1. The flow at the entrance is assumed to be parabolic whereas for the outlet a fully developed flow boundary condition was imposed. The origin of the Cartesian coordinate system is located at the inlet of the geometry, Figure 2.1.

For the fluid (blood) flow the following assumptions are made. Blood is considered to be an electrically conducting biomagnetic Newtonian fluid [14]. The flow is considered to be laminar and the increment of the viscosity due to the magnetic field is considered to be negligible. The walls of the channel are assumed electrically nonconducting and the electric field is considered negligible.

Under the above assumptions the equations governing the flow under consideration are [14, 16]:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \qquad (2.1)$$

$$\bar{\rho}\left(\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right) = -\frac{\partial\bar{p}}{\partial\bar{x}} - \bar{\sigma}\bar{B}^2\bar{u} + \bar{\mu}\left(\frac{\partial^2\bar{u}}{\partial\bar{x}^2} + \frac{\partial^2\bar{u}}{\partial\bar{y}^2}\right),\tag{2.2}$$

$$\bar{\rho}\left(\bar{u}\frac{\partial\bar{v}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{v}}{\partial\bar{y}}\right) = -\frac{\partial\bar{p}}{\partial\bar{y}} + \bar{\mu}\left(\frac{\partial^2\bar{v}}{\partial\bar{x}^2} + \frac{\partial^2\bar{v}}{\partial\bar{y}^2}\right),\tag{2.3}$$

The boundary conditions of the problem are:

$$\begin{array}{ll} \text{Inflow} & (\bar{x}=0,-\bar{D}/2\leq\bar{y}\leq\bar{D}/2): \quad \bar{u}=\bar{u}(\bar{y}), \ \bar{v}=0, \\ \text{Outflow} & (\bar{x}=\bar{L},-\bar{D}/2\leq\bar{y}\leq\bar{D}/2): \quad \partial(\bar{R})/\partial\bar{x}=0, \\ \text{Upper plate} & (\bar{y}=\bar{D}/2,0\leq\bar{x}\leq\bar{L}): \quad \bar{u}=0, \ \bar{v}=0, \\ \text{Lower plate} & (\bar{y}=-\bar{D}/2,0\leq\bar{x}\leq\bar{L}): \quad \bar{u}=0, \ \bar{v}=0. \end{array} \right\}$$
(2.4)

In the equations above  $\vec{q} = (\bar{u}, \bar{v})$  is the dimensional velocity,  $\bar{p}$  is the pressure,  $\bar{u}(\bar{y})$  is a parabolic velocity profile corresponding to the fully developed flow,  $\bar{R}$  stands for  $\bar{u}$  or  $\bar{v}, \bar{\rho}$  is the biomagnetic fluid density,  $\bar{\sigma}$  is the electrical conductivity,  $\bar{\mu}$  is the dynamic viscosity,  $\bar{B}$  is the magnetic induction, and the bar above the quantities denotes that they are dimensional. The term  $\bar{\sigma}\bar{B}^2\bar{u}$  appearing in (2.2), represents the Lorentz force per unit volume and arises due to the electrical conductivity of the fluid [1, 2, 12].

### **3** Transformation of Equations

In order to proceed to the numerical solution of the system (2.1)–(2.3) with the boundary conditions (2.4), the following nondimensional variables are introduced:

$$x = \frac{\bar{x}}{\bar{D}}, \qquad y = \frac{\bar{y}}{\bar{D}}, \qquad u = \frac{\bar{u}}{\bar{u}_0}, \qquad v = \frac{\bar{v}}{\bar{u}_0}, \qquad p = \frac{\bar{p}}{\bar{\rho}\bar{u}_0^2},$$
 (3.1)

where  $\bar{u}_0$  is the maximum velocity of the blood at the entrance of the channel and  $\bar{D}$  is the distance between the two plates at the entrance.

By substitution of (3.1) to equations (2.1)–(2.3), the following system of equations is derived

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} - Mu + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$
(3.3)

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{\operatorname{Re}}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$
(3.4)

The nondimensional parameters entering into the problem under consideration are

$$Re = \frac{\bar{D}\bar{\rho}\bar{u}_0}{\bar{\mu}} \text{ (Reynolds number), } M = \frac{\bar{\sigma}\bar{B}^2\bar{D}}{\bar{u}_0\bar{\rho}} = \frac{Ha^2}{Re} \text{ (Magnetic parameter),}$$

where Ha =  $\frac{\bar{\sigma}\bar{B}^2\bar{D}^2}{\bar{\mu}}$  is the Hartman number. The boundary conditions are now transformed to:

Inflow 
$$(x = 0, -0.5 \le y \le 0.5)$$
:  $u = u(y), v = 0,$   
Outflow  $(x = \bar{L}/\bar{D}, -0.5 \le y \le 0.5)$ :  $\partial(R)/\partial x = 0$   
Upper plate  $(y = 0.5, 0 \le x \le \bar{L}/\bar{D})$ :  $u = 0, v = 0,$   
Lower plate  $(y = -0.5, 0 \le x \le \bar{L}/\bar{D})$ :  $u = 0, v = 0.$ 

$$(3.5)$$

The parameter M, is the magnetic parameter which is the ratio of the square of the Hartman number to the Reynolds Number [1, 2, 12]. It is worth mentioning here that when M is zero, the problem is reduced to the problem of a common hydrodynamic flow in a channel with a stenosis.

# 4 Generalized Curvilinear Coordinate Method and Numerical Approach

A generalized nonorthogonal curvilinear coordinate approach with  $\xi$  and  $\eta$  as independent variables were used to formulate the system of equations for describing complex geometries as the stenosis area in this physical problem [13]. The nondimensional equations can be written as:

$$\frac{\partial U}{\partial \xi} + \frac{\partial V}{\partial \eta} = 0, \tag{4.1}$$

$$\frac{\partial (Uu)}{\partial \xi} + \frac{\partial (Vu)}{\partial \eta} = -\left(y_{\eta}\frac{\partial p}{\partial \xi} - y_{\xi}\frac{\partial p}{\partial \eta}\right) - JMu + \frac{1}{\operatorname{Re}}\frac{\partial}{\partial \xi}\left(q_{1}u_{\xi} - q_{2}u_{\eta}\right) + \frac{1}{\operatorname{Re}}\frac{\partial}{\partial \eta}\left(q_{3}u_{\eta} - q_{2}u_{\xi}\right),$$
(4.2)

$$\frac{\partial (Uv)}{\partial \xi} + \frac{\partial (Vv)}{\partial \eta} = -\left(x_{\xi}\frac{\partial p}{\partial \eta} - x_{\eta}\frac{\partial p}{\partial \xi}\right) + \frac{1}{\operatorname{Re}}\frac{\partial}{\partial \xi}\left(q_{1}v_{\xi} - q_{2}v_{\eta}\right) + \frac{1}{\operatorname{Re}}\frac{\partial}{\partial \eta}\left(q_{3}v_{\eta} - q_{2}v_{\xi}\right),$$
(4.3)

where U, V are the velocity components at  $\xi$  and  $\eta$  directions and the coefficients  $q_1$ ,  $q_2$  and  $q_3$  are given as:

$$U = y_{\eta}u - x_{\eta}v, \quad V = x_{\xi}v - y_{\xi}u,$$
 (4.4)

$$q_1 = \frac{1}{J} \left( x_\eta^2 + y_\eta^2 \right), \quad q_2 = \frac{1}{J} \left( x_\xi y_\eta + y_\xi x_\eta \right), \quad q_3 = \frac{1}{J} \left( x_\xi^2 + y_\xi^2 \right), \tag{4.5}$$

where J is the determinant of the inverse Jacobian of the transformation and  $x_{\xi}$ ,  $x_{\eta}$ ,  $y_{\xi}$ ,  $y_{\eta}$  are the metrics of the transformation.



Figure 4.1: Computational grid at the stenotic region, appropriate stretching of the grid was applied. Close to the walls the grid was finer and became coarser at the center of the domain.

A staggered grid arrangement was further used in our approach, offering advantages over the collocated arrangement especially in convective dominated flows [7]. An appropriate stretching of the grid was applied where the computational grid was finer close to the walls and became coarser close to the center of the domain, Figure 4.1. The upwind scheme is introduced to the discretized equations to overcome problems concerning high convection terms in the momentum. The upwind scheme provides a first order accuracy instead of second order that the diffusion terms retain, leading to inaccurate solution when the local velocity gradients are large. To overcome this problem the "deferred correction" approach was utilized. In this scheme, higher-order flux approximations (central difference scheme) are computed explicitly and this approximation is combined with implicit low-order approximations (upwind difference scheme) [4].

The semi-implicit method for pressure linked equations (SIMPLE) was used to solve the system of the momentum and pressure correction equations. Choosing the under-relaxation factors in SIMPLE method could be challenging, since these factors are problem-specific. In the current study, the relaxation factor used for the momentum equations was equal to 0.7 whereas for the pressure correction equations it was equal to 0.3. More details about the SIMPLE algorithm can be found elsewhere [4, 8].

## 5 Results and Discussion

The above described numerical technique in generalized curvilinear coordinates was applied to solve the system of equations (3.2)–(3.4), under the appropriate boundary conditions (3.5). To proceed to the derivation of the numerical results, it is necessary to assign values to the dimensionless parameters entering into the problem under consideration,



Figure 5.1: Effect of magnetic field on the dimensionless velocity at two different locations for various magnetic field intensities: (A) At the center of the stenosis. (B) Downstream the stenosis by one diameter  $\overline{D}$  from the center of the stenosis.

such as the Reynolds number, Re, and the magnetic parameter, M. Re was assumed to be equal to 400, which corresponds to the peak systolic Re in a coronary artery. The electrical conductivity  $\bar{\sigma}$  of stationary blood was measured to be 0.7  $Sm^{-1}$  [6].

The electrical conductivity of flowing blood is always greater than that of the stationary. The increment for medium shear rates is about 10% and increases with the increment of the hematocrit [5]. In the current study the electrical conductivity of blood was assumed, for simplicity, temperature–independent, and equal to  $0.8 \text{ Sm}^{-1}$  as in [14]. Four different values of the magnetic induction,  $\overline{B}$ , were studied, ( $\overline{B} = 0, 8, 16$  and 32 T), where for  $\overline{B} = 0$  we have the hydrodynamic case. These values of magnetic induction correspond to magnetic parameter M = 0 for the hydrodynamic case and M = 36.6, 146.3, 585.1 for the cases where  $\overline{B} = 8, 16, 32 T$ , respectively and for  $\bar{\mu} = 0.0035 \text{ kg/ms}$ . Figure 5.1 shows the effect of the magnetic field and the magnetic parameter on the dimensionless velocity at two different locations in the computational domain, at the center of the 50% stenosis, location A, and downstream of the stenosis by one diameter D from the center of the stenosis, location B (Figure 2.1). The results showed that the dimensionless velocity magnitude does not change substantially at location A with the increase of the magnetic field. However, downstream of the stenosis, location B, application of the magnetic field results in a substantial reduction of the dimensionless velocity magnitude and a reduction of the flow reversal. These findings are very important and suggest a significant flow reduction alleviating the hydrodynamic stresses downstream of the artery stenosis.

These results are highlighted in Figure 5.2. The figure depicts the effect of the magnetic field on the recirculation zone downstream of the stenosis. It is apparent that the increase of the magnetic field substantially reduces the recirculation zone leading to a reduction of the hydrodynamic stresses at this region. One of the most important flow characteristics is the local skin friction coefficient,  $C_f$ . This quantity can be defined by



Figure 5.2: Effect of magnetic field on the dimensionless velocity and recirculation blood zones downstream of the stenosis.

Mag. intensity $\bar{B}$	$C_f$ at loc. A	$C_f$ at loc. B	$u_{max}$	$\Delta p$
0 T	0.273	0.15	1.536	0.8
8 T	0.277 (1.5 %)	0.145 (-3.3 %)	1.524 (-0.8%)	0.9
16 T	0.288 (5.5 %)	0.13 (-13.3 %)	1.494 (-2.7%)	1.6
32 T	0.313 (14.7 %)	0.025 (-83.3 %)	1.457 (-5.1%)	4.6

Table 5.1: Local skin friction coefficient,  $C_f$ , maximum dimensionless velocity,  $u_{max}$  and dimensionless pressure drop,  $\Delta p$ , for hydrodynamic and MHD cases.

the following relation:

$$C_f = \frac{2\bar{\tau}_w}{\bar{\rho}\bar{u}_0^2},\tag{5.1}$$

where  $\bar{\tau}_w = \bar{\mu} \left( \partial \bar{u} / \partial \bar{y} \right)|_{\bar{y} = -\bar{D}/2, \bar{D}/2}$  is the wall shear stress between the fluid and the plates. Table 5.1 summarizes the local skin friction coefficient,  $C_f$ , variation at the two different locations of Figure 5.1. It is observed that the local skin friction coefficient,  $C_f$ , increases at location A (center of the stenosis) but substantially decreases downstream of the stenosis at location B due to the magnetic field applied uniformly throughout the stenosed channel. Table 5.1 also reports the reduction of the maximum velocity magnitude due to the magnetic field effect.

The results concerning the velocity and skin friction coefficient show that the flow is appreciably influenced by the magnetic field. A substantial reduction of the vortices formed downstream of the stenosis was observed, significantly affecting the flow characteristics. However, the pressure drop in the domain significantly increased as the magnetic parameter increased. This pressure increment is due to the magnetohydrodynamic pressure induced to the flow field from the magnetic field as depicted in Table 5.1. This pressure drop increase requires further investigation and it would be a next step of this study. The introduction of the energy equation would be also a future step of this study. The energy equation could give important information about the temperature increase of the biofluid (blood) due to the magnetic parameter increase.

Vulnerable plaque rupture usually occurs towards the luminal side of the plaque and is dependent on the fibrous cap thickness and presence of microcalcifications [10]. Since peak velocities, and therefore stresses, are observed in the stenotic region as depicted in Figure 5.1A, reduction in downstream stresses (Figure 5.1B) may not mitigate the risk of rupture of the fibrous cap. On the other hand the reduction in the size of the recirculation zones under a magnetic field may help reduce the incidence of thrombosis in the stenosis region. The potential MHD effects on blood flowing through the region of interest could be a small change in viscosity or even platelet deposition and cell damage. However, these effects require further experimental investigation beyond the scope of this study.

The above results indicate that application of a magnetic field on the flow of a

biomagnetic fluid could be useful for medical and engineering applications leading to a new avenue of flow control in stenotic regions.

### 6 Concluding Remarks

The biomagnetic fluid (blood) flow in a channel with symmetric stenosis is studied. The numerical solution of the problem is obtained by the development of a numerical technique based on the finite volumes discretization scheme in curvilinear coordinates for the discretization of the nonlinear system of PDEs governing the MHD blood flow. This methodology is based on the development of a semi-implicit numerical technique, transformations, stretching of the grid, and proper construction of the boundary conditions on the solid walls. The proposed model can predict the reduction of blood's velocity field and the change of the recirculation zones downstream of the stenosis. The skin friction coefficient,  $C_f$ , was locally increased at the stenosis area and significantly reduced at the recirculation zones as the magnetic field intensity was increased. The proposed model could predict pathological conditions such as the stenotic coronary arteries and give the opportunity for new avenues of flow control at these stenotic regions without the need of in vivo testing of the patient.

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