Three-Dimensional Magnetic Fluid Boundary Layer Flow Over a Linearly Stretching Sheet

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The three-dimensional laminar and steady boundary layer flow of an electrically nonconducting and incompressible magnetic fluid, with low Curie temperature and moderate saturation magnetization, over an elastic stretching sheet, is numerically studied. The fluid is subject to the magnetic field generated by an infinitely long, straight wire, carrying an electric current. The magnetic fluid far from the surface is at rest and at temperature greater of that of the sheet. It is also assumed that the magnetization of the fluid varies with the magnetic field strength H and the temperature T. The numerical solution of the coupled and nonlinear system of ordinary differential equations, resulting after the introduction of appropriate nondimensional variables, with its boundary conditions, describing the problem under consideration, is obtained by an efficient numerical technique based on the common finite difference method. Numerical calculations are carried out for the case of a representative water-based magnetic fluid and for specific values of the dimensionless parameters entering into the problem, and the obtained results are presented graphically for these values of the parameters. The analysis of these results showed that there is an interaction between the motions of the fluid, which are induced by the stretching surface and by the action of the magnetic field, and the flow field is noticeably affected by the variations in the magnetic interaction parameter β. The important results of the present analysis are summarized in Sec. 6.
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Keywords: FHD, magnetic fluid, three-dimensional flow, stretching sheet, low Curie temperature, moderate saturation magnetization

1 Introduction

The investigation of the two-dimensional flow problems, caused by a linearly stretching flat surface, in an otherwise quiescent incompressible fluid, has been proved of fundamental importance in recent years due to their applications in a number of mechanical and technological processes, and several authors have studied various aspects of this problem [1–10]. More specifically, the generalized three-dimensional boundary layer flow of a viscous and in an otherwise ambient incompressible viscous fluid, due to a stretching sheet, was also studied by Wang [11], Ariel [12], and Takhar et al. [13].

On the other hand, during the last decades, there is a steady growth in the study and applications of colloidal stable magnetizable fluids. The new scientific advances and the increasing importance of the technological applications of magnetic fluids, and especially of ferromagnetic ones, have led to widely dispersed growing research groups of many nationalities all over the world. The behavior of a ferrofluid, under the action of an applied magnetic field, is of fundamental importance not only in ferrohydrodynamics (FHD) but also in biomagnetic fluid dynamics (BFD) in which blood is investigated as a magnetic fluid. So, an extensive work has been done in this field, and some representative works can be found in Refs. [14–25].

However, it is known that a magnetic fluid, also known as a ferrofluid, is a suspension of ferromagnetic or ferrimagnetic particles in a carrier liquid (water-based or oil-based ferrofluids). In certain applications, such as in energy conversion devices, it is necessary to use a fluid with large pyromagnetic coefficient K, i.e., with a high saturation magnetization and a low Curie temperature T_c. So, the above mentioned magnetic particles are unsuitable for this purpose since they have Curie temperatures higher than the boiling point of the carrier liquid. To overcome this difficulty, some investigators [26–32] have synthesized fine particles of different types and obtained water-based magnetic fluids, such as EMG 901, EMG 909, and EMG 805, with large pyromagnetic coefficient K and moderately low curie temperatures T_c.

So, the purpose of the present work is to study the three-dimensional laminar and steady boundary layer flow of an electrically nonconducting and incompressible magnetic fluid, with low Curie temperature and moderate saturation magnetization, over a stretching sheet. The fluid is subject to the magnetic field generated by an infinitely long, straight wire oriented along the x-axis of a Cartesian coordinate system $Oxyz$. The wire carries an electric current I and it is placed parallel to the stretching surface (Oxy-plane) at a distance d below it. The magnetic fluid, far from the surface, is initially at rest and at temperature greater of that of the sheet. This physical problem concerns the FHD flow, but it can also be considered as a BFD flow as long as the conditions referred in Ref. [14] hold. It is also assumed that the magnetization varies with the magnetic field strength H and the temperature T according to the relation

$$M = KH(T_c - T) \tag{1}$$

and this relation is the one derived experimentally in Ref. [25], suggested in Ref. [14], and also used in Ref. [33].

The formulation of the problem is obtained by an analogous manner presented in Refs. [23,24], and the numerical solution is obtained by applying an efficient numerical technique based on the common finite difference method [34,35]. The obtained numerical results for a representative water-based magnetic fluid, with moderate saturation magnetization and low Curie tempera-

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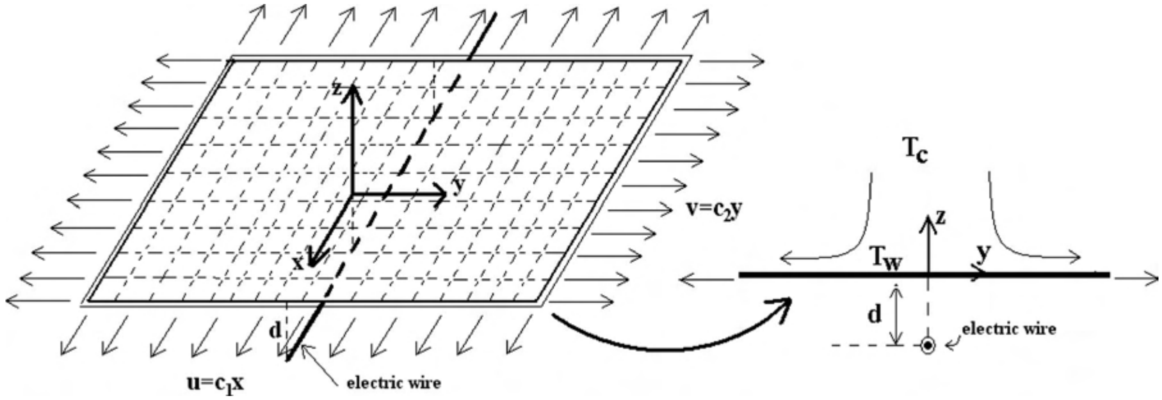

Fig. 1 Schematic representation of flow configuration

ture, are presented graphically for specific values of the parameters entering into the problem under consideration and analyzed in detail.

2 Mathematical Formulation

The steady, three-dimensional, incompressible, laminar boundary layer magnetic fluid flow, induced by the stretching of a highly elastic flat surface in two lateral directions, in an otherwise quiescent fluid is considered. The stretching surface is placed in the plane $z=0$, whereas the fluid occupies the upper half plane $z\geq 0$. The velocities of the stretching surface in the x- and y-directions, respectively, and therefore of the fluid in contact with it, and in a Cartesian coordinate system $Oxyz$, are given by

$$u=c_1x, \quad v=c_2y, \quad w=0 \tag{2}$$

where c_1 and c_2 are dimensional constants. The fluid temperature, far away from the sheet, is T_c, whereas the stretching surface is kept at a constant temperature T_w, less than T_c. The viscous and electrically nonconducting magnetic fluid is subject to the action of a magnetic field $\boldsymbol{H}$, which is generated by an electric current with intensity I. The electric current is going through an infinite thin wire placed parallel to the x-axis and at a distance d below it. The electric current in the wire flows from the positive x-axis toward the negative, and a schematic representation of this flow configuration is presented in Fig. 1.

It is also assumed that this electric current in the wire gives rise to a magnetic field of sufficient strength to saturate the magnetic fluid so that the equilibrium magnetization is attained.

Under the above assumptions the equations governing the physical problem under consideration are the mass conservation, fluid momentum in the x-, y-, and z-directions, and energy equation and can be written as [14,15,18,23,24,33]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{3}$$

$$\rho\left(u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right) \tag{4}$$

$$\rho\left(v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+\mu_0M\frac{\partial H}{\partial y} \tag{5}$$

$$\rho w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\mu\frac{\partial^2 w}{\partial z^2}+\mu_0M\frac{\partial H}{\partial z} \tag{6}$$

$$\rho c_p\boldsymbol{q}\,.\,(\boldsymbol{\nabla}T)+\mu_0T\frac{\partial M}{\partial T}\left(v\frac{\partial H}{\partial y}+w\frac{\partial H}{\partial z}\right)=k\nabla^2T+\mu\Phi \tag{7}$$

In the above equation (7), Φ is the dissipation function, which, in the case under consideration, is given by the expression

$$\Phi=2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]+\left[\left(\frac{\partial v}{\partial z}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2\right] \tag{8}$$

The system of Eqs. (3)–(7) is subject to the following boundary conditions:

$$z=0: \quad u=c_1x, \quad v=c_2y, \quad w=0, \quad T=T_w \tag{9}$$

$$z\to\infty: \quad u=0, \quad v=0, \quad T=T_c, \quad p+\rho\tfrac{1}{2}w^2=p_\infty=\text{const} \tag{10}$$

The terms $\mu_0M(\partial H/\partial y)$ and $\mu_0M(\partial H/\partial z)$ in Eqs. (5) and (6), respectively, represent the components of the magnetic force, per unit volume of the fluid, and depend on the existence of the magnetic gradient. When the magnetic gradient is absent these forces vanish. The second term, on the left-hand side of the energy equation (6), accounts for heating due to the adiabatic magnetization.

The components H_y, H_z of the magnetic field $\boldsymbol{H}=(H_y, H_z)$, due to the electric current flowing through the wire with intensity I, are given by

$$H_y(y,z)=-\frac{I}{2\pi}\frac{z+d}{y^2+(z+d)^2} \tag{11}$$

$$H_z(y,z)=\frac{I}{2\pi}\frac{y}{y^2+(z+d)^2} \tag{12}$$

Therefore, the magnitude $\|\boldsymbol{H}\|=H$, of the magnetic field, is given by

$$H(x,y,z)\equiv H(y,z)=[H_y^2+H_z^2]^{1/2}=\frac{I}{2\pi}\frac{1}{\sqrt{y^2+(z+d)^2}} \tag{13}$$

It is reminded, once more, that when the applied magnetic field $\boldsymbol{H}$ is sufficiently strong to saturate the magnetic fluid, the magnetization M is, generally, determined by the fluid temperature T and the magnetic field strength H and, in the problem under consideration, it is expressed by relation (1), i.e., $M=KH(T_c-T)$.

3 Transformation of Equations

The mathematical analysis of the problem under study is simplified by introducing the following dimensionless coordinates:

$$\xi(x)=\left(\frac{c}{\nu}\right)^{1/2}x, \quad \zeta(y)=\left(\frac{c}{\nu}\right)^{1/2}y, \quad \eta(z)=\left(\frac{c}{\nu}\right)^{1/2}z \tag{14}$$

where $c=(c_1+c_2)/2$.

In addition, the corresponding velocity components of the fluid, defined by the expressions

$$u(\xi,\eta)=\sqrt{c\nu}\xi f'(\eta),\quad v(\zeta,\eta)=\sqrt{c\nu}\zeta g'(\eta),$$
$$w(\eta)=-\sqrt{c\nu}[f(\eta)+g(\eta)] \tag{15}$$

where $(\ )'=\partial(\ )/\partial\eta$ satisfy the continuity equation (3).

Finally, the dimensionless pressure (magnetic and static) $P(\xi,\zeta,\eta)$ and temperature $\Theta(\xi,\zeta,\eta)$ of the magnetic fluid are given by the following expressions:

$$P(\xi,\zeta,\eta)=\frac{p}{c\mu}=P_1(\eta)+\xi P_2(\eta)+\xi^2P_3(\eta)+\zeta P_4(\eta)+\zeta^2P_5(\eta)+\xi\zeta P_6(\eta) \tag{16}$$

$$\Theta(\xi,\zeta,\eta)=\frac{T_c-T}{T_c-T_w}=\Theta_1(\eta)+\xi\Theta_2(\eta)+\xi^2\Theta_3(\eta)+\zeta\Theta_4(\eta)+\zeta^2\Theta_5(\eta)+\xi\zeta\Theta_6(\eta) \tag{17}$$

As far as the magnetic field is concerned, the magnitude H of the magnetic field strength is given by the expression

$$H(\zeta,\eta)=\frac{I}{2\pi}\sqrt{\frac{c}{\nu}}\frac{1}{\sqrt{\zeta^2+(\eta+\alpha)^2}} \tag{18}$$

where α is the dimensionless distance of the electric wire from the ξ-axis ($\alpha=d\sqrt{c/\nu}$).

For the calculation of the partial derivatives of the above expression, with respect to ζ and η, a similar procedure to that first used in Refs. [23,24] is followed. First, Eq. (18) is expanded in powers of ζ up to terms of the order of ζ^2. Hereafter, the corresponding derivatives are calculated by partial differentiation of the resulting expansion already mentioned. So, the partial derivatives of the magnetic field, with respect to ζ and η, are given by the following expressions:

$$\frac{\partial H}{\partial\zeta}=-\frac{c}{\nu}\frac{I}{2\pi}\frac{\zeta}{(\eta+\alpha)^3} \tag{19}$$

$$\frac{\partial H}{\partial\eta}=-\frac{c}{\nu}\frac{I}{2\pi}\frac{1}{(\eta+\alpha)^2}\left[1-\frac{3}{2}\frac{\zeta^2}{(\eta+\alpha)^2}\right] \tag{20}$$

By substituting Eq. (1) and all the above expressions (14)–(20) into the momentum equations (4)–(6), and into the energy equation (7), and following standard procedure, the problem under consideration is finally described by the following system of ordinary differential equations:

$$f'''+(f+g)f''-(f')^2-2P_3=0 \tag{21}$$

$$g''+(f+g)g''-(g')^2-2P_5-\frac{\beta\Theta_1}{(\alpha+\eta)^4}=0 \tag{22}$$

$$P_1'+f''+g''+(f+g)(f'+g')+\frac{\beta\Theta_1}{(\alpha+\eta)^3}=0 \tag{23}$$

$$P_3'+\frac{\beta\Theta_3}{(\alpha+\eta)^3}=0 \tag{24}$$

$$P_5'-\frac{2\beta\Theta_1}{(\alpha+\eta)^5}+\frac{\beta\Theta_5}{(\alpha+\eta)^3}=0 \tag{25}$$

$$\Theta_1''+\Pr(f+g)\Theta_1'+\frac{\lambda\beta(\Theta_1-\varepsilon)}{(\alpha+\eta)^3}(f+g)+2(\Theta_3+\Theta_5)-4\lambda[(f')^2+(g')^2+f'g']=0 \tag{26}$$

$$\Theta_3''+\Pr(f+g)\Theta_3'+\left[\lambda\beta\frac{f+g}{(\alpha+\eta)^3}-2\Pr f'\right]\Theta_3-\lambda(f'')^2=0 \tag{27}$$

$$\Theta_5''+\Pr(f+g)\Theta_5'+\left[\lambda\beta\frac{f+g}{(\alpha+\eta)^3}-2\Pr g'\right]\Theta_5+\beta\lambda(\varepsilon-\Theta_1)\times\left[\frac{2(f+g)}{(\alpha+\eta)^5}+\frac{g'}{(\alpha+\eta)^4}\right]=0 \tag{28}$$

subject to the boundary conditions

$$\eta=0:\quad f'=\delta_1,\quad g'=\delta_2,\quad f=g=0,\quad \Theta_1=1,\quad \Theta_3=\Theta_5=0 \tag{29}$$

$$\eta\to\infty:\quad f'=g'=0,\quad P_1\to P_\infty,\quad P_3=P_5=0,\quad \Theta_1=\Theta_3=\Theta_5=0 \tag{30}$$

The dimensionless parameters appearing in these equations are defined as follows:

$$\Pr=\mu c_p/k\quad\text{(Prandtl number)}$$

$$\lambda=\frac{c\mu^2}{\rho k(T_c-T_w)}\quad\text{(viscous dissipation parameter)}$$

$$\varepsilon=T_c/(T_c-T_w)\quad\text{(dimensionless temperature parameter)}$$

$$\alpha=(c\rho/\mu)^{1/2}d\quad\text{(dimensionless distance)}$$

$$\beta=\frac{I^2}{4\pi^2}\frac{K\mu_o(T_c-T_w)\rho}{\mu^2}\quad\text{(dimensionless magnetic interaction parameter)} \tag{31}$$

and the quantities δ_1 and δ_2 are also defined as

$$\delta_1=c_1/c\quad\text{and}\quad\delta_2=c_2/c\quad\text{where } c=(c_1+c_2)/2 \tag{32}$$

For equal speeds of stretching in the ξ- and ζ-directions, it is $c_1=c_2=c$ and therefore the corresponding boundary conditions for the functions f' and g' become $f'(0)=\delta_1=1$ and $g'(0)=\delta_2=1$. It is worth noting, however, that other cases could also be considered for these boundary conditions. For instance, when $c_1=0$ and $c_2=2c$ there is stretching only toward the ζ-direction (two-dimensional flow) and the corresponding boundary conditions become $f'(0)=\delta_1=0$ and $g'(0)=\delta_2=2$. However, in the problem under consideration, and due to the lack of space, only the most representative case $c_1=c_2=c$ is examined.

The system of Eqs. (21)–(28), subject to the boundary conditions (29) and (30), is a five-parameter $(\alpha,\beta,\lambda,\varepsilon,\Pr)$ coupled and nonlinear system of ordinary differential equations, describing the magnetic fluid flow over the stretching sheet when the magnetization of the fluid is given as a function of temperature T and magnetic field strength H.

4 Numerical Solution Method

For the numerical solution of the system of ordinary differential equations, of the problem under consideration, and for specific values of the dimensionless parameters appearing in it, a numerical technique is applied with the following characteristics. It is based on the common finite difference method with central differencing, a tridiagonal matrix manipulation, and an iterative procedure. The whole numerical scheme can be programed and applied easily, is stable, accurate, and rapidly converging. This solution methodology, for a system of three equations, is described in detail in Refs. [34,35]. It has also been extended, applied, and validated in Refs. [24,33].

In order to obtain and present numerical results, for the problem under consideration, and analyze them, the following assumptions are made for the magnetic fluid and the values of the dimensionless parameters, entering into the problem and defined in Eq. (31).

(a) The magnetic fluid is similar to that used in Ref. [32] (a water-based magnetic fluid Ferrotec™ EMG 805, 300 G, with density $\rho = 1180\ \mathrm{kg/m^3}$, kinematical viscosity $\nu = 1.69 \times 10^{-6}\ \mathrm{m^2/s}$, coefficient of thermal diffusivity $a = 0.119 \times 10^{-6}\ \mathrm{m^2/s}$, and saturation magnetization $M_s = 15.61 \times 10^3\ \mathrm{A/m}$), but with low Curie temperature and moderate saturation magnetization. The Prandtl number of this fluid is equal to $\mathrm{Pr} = 14.20$.

(b) The temperature T_w of the stretching sheet is taken equal to 280 K, whereas the fluid temperature far away from the sheet is taken equal to $T_c = 340$ K. In such a case the dimensionless temperature parameter ε is equal to $\varepsilon = 5.667$.

(c) The dimensional constants c_1 and c_2 express the rate of stretching of the elastic sheet, per unit length, along the x- and y-axes, respectively. For equal speeds of stretching in x- and y-directions (or in ξ- and ζ-directions) it is taken as $c_1 = c_2 = c = 1$ and in such a case, the boundary conditions for the functions f' and g' become $f'(0) = \delta_1 = 1$ and $g'(0) = \delta_2 = 1$.

(d) The infinitely long and straight current carrying wire is placed at a distance $d = 0.01$ m below the x-axis and parallel to it. In such a case, the dimensionless distance a, defined by the relation $\alpha = (c/\nu)^{1/2} d$, is equal to $a = 7.69$.

(e) According to its definition and to the physical properties of the fluid under consideration, the viscous dissipation parameter λ is equal to $\lambda = 9.5234 \times 10^{-11}$.

(f) Finally, the dimensionless magnetic interaction parameter β, defined as $\beta = (I^2/4\pi^2) K \mu_o (T_c - T_w) \rho / \mu^2$, can also be defined as $\beta = M_s B_s a^2 / c\mu$, where $M_s = H(0,0) K (T_c - T_w)$, $B_s = \mu_0 H(0,0)$, and $H(0,0)$ is the value of the magnetic field strength for $\zeta = \eta = 0$, that is, $H(0,0) = (I/2\pi) \cdot 1/d$.

According to the assumptions made for the properties of the magnetic fluid and the flow configuration, a plausible value for the dimensionless magnetic interaction parameter β that saturates the fluid is $\beta = 1.5 \times 10^5$. It is worth reminding here that the case $\beta = 0.0$ corresponds to the hydrodynamic flow.

For these values of the dimensionless parameters, entering into the problem under consideration, the obtained numerical results, concerning the dimensionless velocity field, temperature field, pressure, skin friction, and rate of heat transfer coefficients, are shown in Figs. 2–10 and analyzed in Sec. 5.

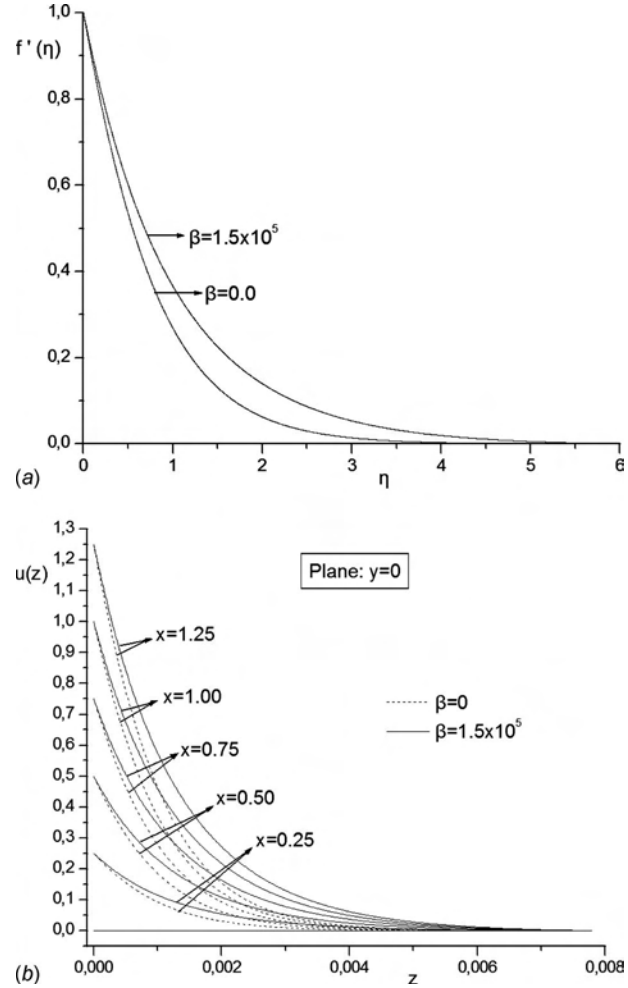

Fig. 2 **(*a*) Dimensionless velocity component $f'(\eta)$ and (*b*) dimensional velocity component $u(z)$ at the plane $y=0$ and for different positions of x**

5 Analysis of the Results

The velocity components of the fluid, along the axes x, y, and z, respectively, are defined by expressions (15):

$$u(\xi,\eta) = \sqrt{c\nu}\,\xi f'(\eta), \quad \upsilon(\zeta,\eta) = \sqrt{c\nu}\,\zeta g'(\eta),$$

$$w(\eta) = -\sqrt{c\nu}[f(\eta) + g(\eta)]$$

It is clear that for a specific vertical plane, parallel, for instance, to the Oyz, e.g., $x = \text{const}$ or $\xi = \text{const}$, the u-velocity component is a function only of the dimensionless variable $\eta = \eta(z)$. The same is true for the υ-velocity component, whereas the w-velocity component is a function only of the dimensionless variable η. So, the variations in the dimensionless velocity components

$$f'(\eta) = u(\xi,\eta)/\sqrt{c\nu}\xi, \quad g'(\eta) = \upsilon(\zeta,\eta)/\sqrt{c\nu}\zeta$$
$$\text{and} \quad -[f(\eta) + g(\eta)] = w(\eta)/\sqrt{c\nu} \tag{33}$$

against the dimensionless distance η from the surface of the stretching sheet are shown in Figs. 2–4, respectively. It is observed that inside the boundary layer over the stretching sheet, the velocity components $f'(\eta)$ and $g'(\eta)$ present the classical behavior of the velocity field taking their limiting value of zero far away from the stretching surface (Figs. 2 and 3). It is worth noting, however, that when the fluid is saturated by the applied magnetic field ($\beta \neq 0.0$) and inside the boundary layer ($0 < \eta < \eta_\infty = 6.0$), the velocity component $f'(\eta)$ is greater than the corresponding one in hydrodynamics case (Fig. 2). However, the opposite is true for the velocity component $g'(\eta)$ (Fig. 3). This fact is due to the influence of the Kelvin force on the flow field in the y- and z-directions. This influence is also evident in Fig. 4, which presents the variations in the dimensionless velocity component in the z-direction. In the ferromagnetic case ($\beta \neq 0.0$) the components of magnetic force per unit volume act in the negative y-direction and in the positive z-direction, respectively, and oppose to the fluid motion that comes as a result of the stretching of the elastic surface. So, there is an interaction between the motions of the fluid that are induced by the stretching surface and by the action of the magnetic field. It is worth noting that in the ferromagnetic case and very close to the elastic sheet ($\eta \cong 0.5$) the velocity component in the y-direction ($g'(\eta)$) takes negative values, which means that in that region the flow is reversed. Quantitatively, when $\eta = 1.0$, for instance, and β increases from 0.0 to 1.5×10^5, the percentage changes (decrements) of the dimensionless velocity components $g'(\eta)$ (Fig. 3) and $-[f(\eta) + g(\eta)]$ (Fig. 4) are 138.75% and 42. 8%, respectively.

The interaction between the motions of the fluid that are induced by the stretching surface and by the action of the magnetic field is also shown in Figs. 2(*b*) and 3(*b*). These figures present the variations in the dimensional velocity components $u(z)$ and $\upsilon(z)$ against z at different distances along the x- and y-axes, respectively. It is concluded that near the surface the magnetic field interacts with the fluid motion that comes as a result of the stretch-

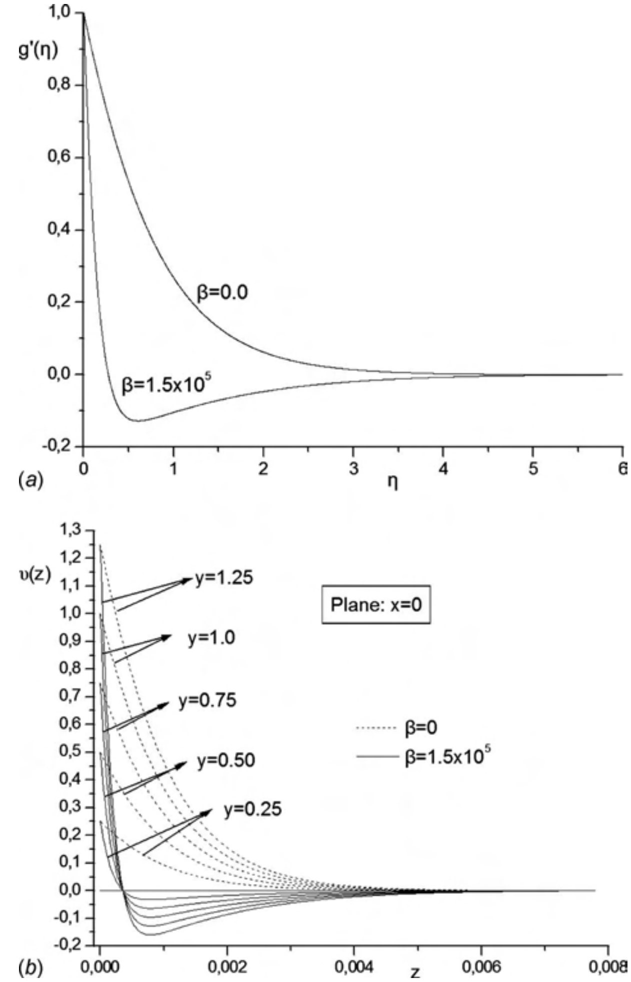

Fig. 3 (*a*) Dimensionless velocity component $g'(\eta)$ and (*b*) dimensional velocity component $\upsilon(z)$ at the plane $x=0$ and for different positions of y

ing of the elastic surface. This interaction increases the $u(z)$ velocity component but decreases the $\upsilon(z)$ velocity component. This different influence of the magnetic field on the velocity components u and υ is due to the orientation of the infinite electrical wire that is placed parallel to the x-axis.

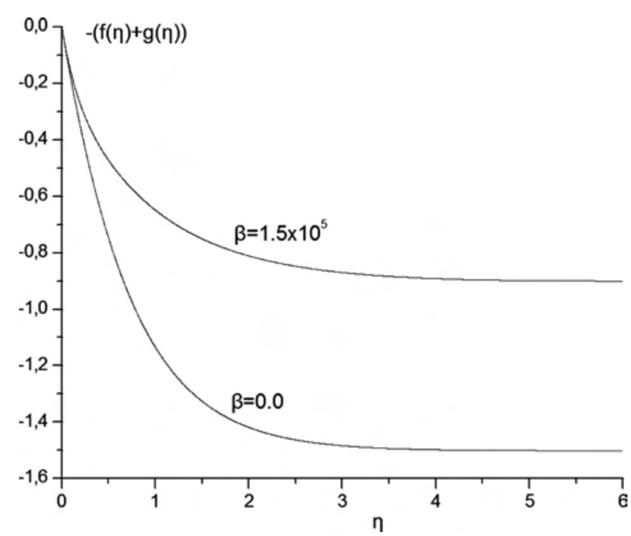

Fig. 4 Dimensionless velocity component $-[f(\eta)+g(\eta)]$

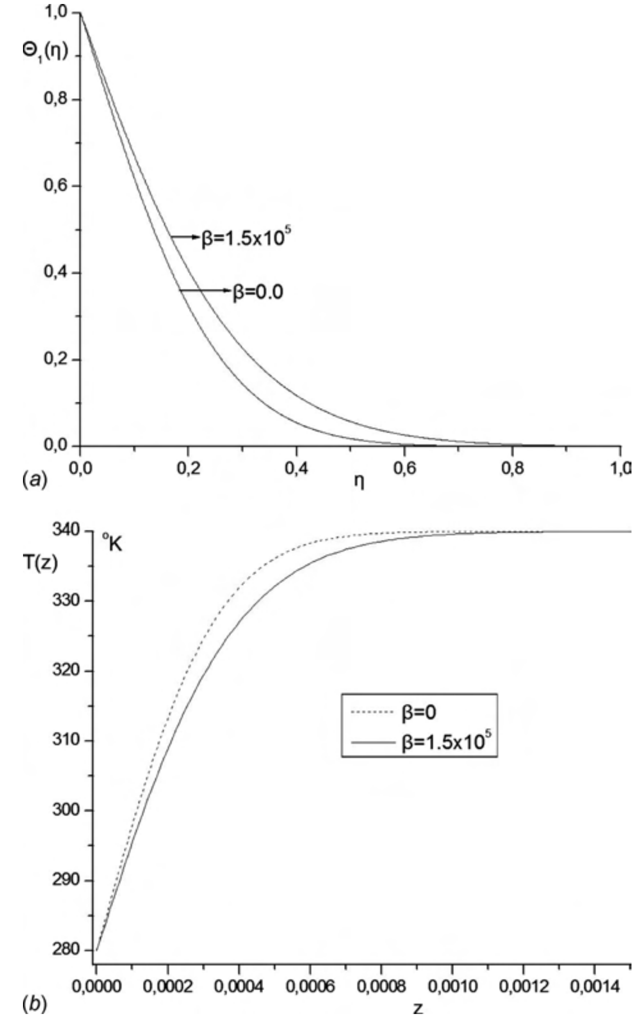

Fig. 5 (*a*) Dimensionless fluid temperature $\Theta_1(\eta)$ and (*b*) dimensional fluid temperature $T(z)$

The numerical analysis and the obtained numerical results showed that the dimensionless temperature $\Theta(\xi,\zeta,\eta)$ of the fluid is represented only by the dimensionless temperature $\Theta_1(\eta)$, e.g., $\Theta_i=0$, for $i=2,\ldots,5$. So, the dimensional temperature T can be expressed, according to Eq. (17), as

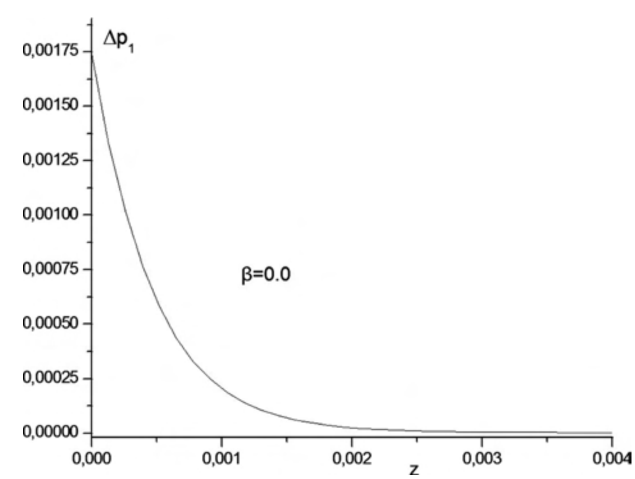

Fig. 6 Dimensional fluid pressure difference Δp_1 in the hydrodynamic case

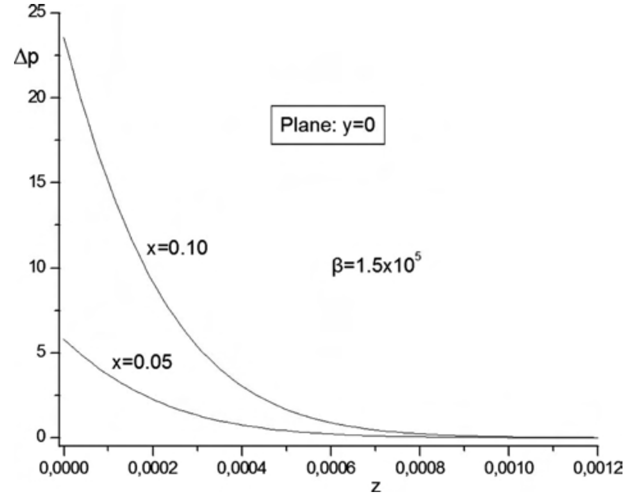

Fig. 7 Dimensional fluid pressure difference Δp in the ferromagnetic case at the plane $y=0$ and for different positions of x

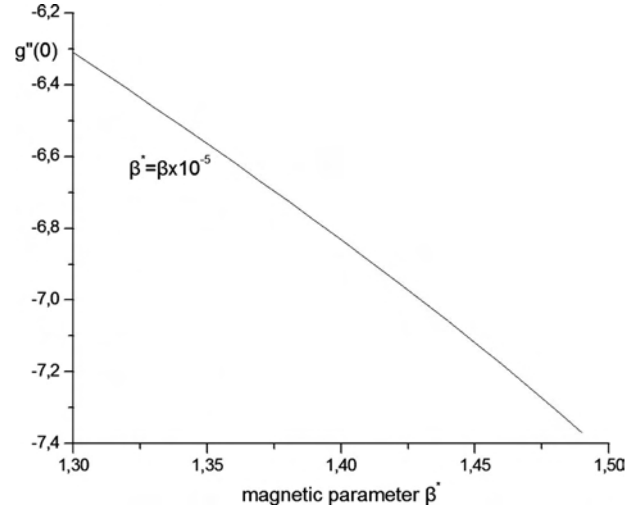

Fig. 9 Dimensionless skin friction coefficient $g''(0)$

$$T(z) = T_c - (T_c - T_w)\Theta_1(\eta(z)) \tag{34}$$

Figure 5 shows the variations in the dimensionless temperature $\Theta_1(\eta)$ in the hydrodynamic case ($\beta=0.0$) as well as in the ferromagnetic one ($\beta=1.5\times10^5$). From this figure it is concluded that the thermal boundary layer thickness $\delta_{\rm th}(\delta_{\rm th}\sim 0.8)$ is much smaller than the corresponding viscous boundary layer thickness $\delta_{\rm vis}(\delta_{\rm vis}\sim 5.0$, and this is in agreement with the relation that compares these thicknesses with Prandtl number $\Pr(\sqrt{\Pr}\sim \delta_{\rm vis}/\delta_{\rm th})$. It is also concluded, from this figure, that the dimensionless temperature $\Theta_1(\eta)$ is greater in the ferromagnetic case than the corresponding one in the hydrodynamic case. However, this increment is not important since, for instance, for $\eta=0.2$, the percentage increment is only 1.2%. However, for the dimensional temperature $T(z)$ the opposite is true. The variation in the dimensional temperature $T(z)$ against the normal distance z, from the elastic surface, for $\beta=0$ (hydrodynamic case) and for $\beta=1.5\times10^5$ (ferromagnetic case) is shown in Fig. 5(*b*). It is concluded, from this figure, that the fluid temperature inside the thermal boundary layer is greater in the hydrodynamic case than that in the ferromagnetic one. So, the contribution, in the increase in fluid temperature, of the second term, on the left-hand side of the energy equation (6), which accounts for heating due to the adiabatic magnetization, cannot counterbalance the decrement of temperature due to the variation in the velocity field in the ferromagnetic case. This was also verified by additional numerical calculations in which the contribution of this term was found to be not important.

The numerical analysis also showed that the dimensionless pressure $P(\xi,\zeta,\eta)$ is represented only by the function $P_1(\eta)$ in the hydrodynamic case and by the functions $P_1(\eta)$ and $P_3(\eta)$ in the ferromagnetic one. So, according to Eq. (16), the dimensional pressure difference $\Delta p=p-p_\infty$ is expressed as

$$\Delta p_1(z) = p(z) - p_\infty = c\mu P_1(\eta(z)) \tag{35}$$

in the hydrodynamic case and as

$$\Delta p(x,z) = p(x,z) - p_\infty = c\mu\left\{P_1(\eta(z)) + \frac{c}{\nu}x^2 P_3(\eta(z))\right\} \tag{36}$$

in the ferromagnetic one.

The variations in these functions are presented in Figs. 6 and 7, respectively. It is concluded that in the hydrodynamic case as well as in the ferromagnetic one, the dimensional pressure differences decrease to zero (or $p\to p_\infty$) as the distance from the stretching surface approaches infinity.

It is worth reminding here that in the ferromagnetic case by the term "pressure" the sum of the static and magnetic pressure of the

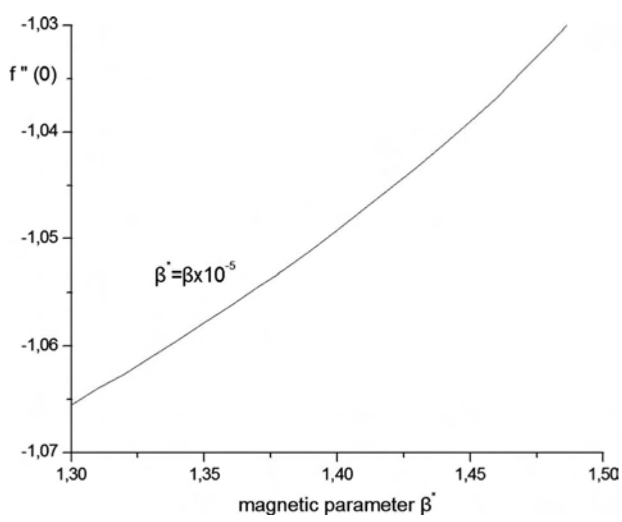

Fig. 8 Dimensionless skin friction coefficient $f''(0)$

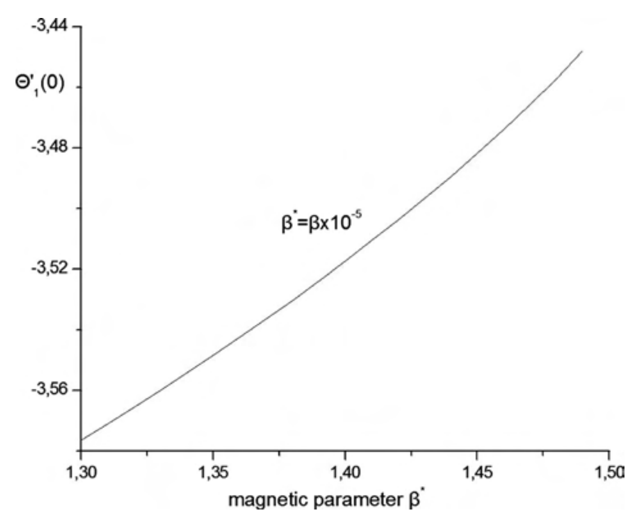

Fig. 10 Dimensionless wall heat transfer coefficient $\Theta_1'(0)$

fluid is insinuated. In such a case the magnetic pressure represents the energy density (internal energy per unit volume of the fluid) associated with the induced magnetic field by the electric current of the wire. Isolation of the static or magnetic pressure is not an easy task and requires modification of the initial governing equations of the mathematical model of FHD.

For the physical problem under study the shearing stresses and the rate of heat transfer, at the stretching elastic surface, are defined as follows:

$$(\tau_{zx})_w = \mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)_{z=0}, \quad (\tau_{zy})_w = \mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)_{z=0} \tag{37}$$

for the shearing stresses in the x- and y-directions, respectively, and

$$\dot{q}_w = -k\left(\frac{\partial T}{\partial z}\right)_{z=0} \tag{38}$$

for the rate of heat transfer at the sheet.

Using transformations (14), (15), and (17) the corresponding dimensionless quantities C_{f_ξ}, C_{f_ζ}, and Nu can be written as

$$C_{f_\xi} = f''(0) = \frac{(\tau_{zx})_w}{\mu c \xi}, \quad C_{f_\zeta} = g''(0) = \frac{(\tau_{zy})_w}{\mu c \zeta},$$

$$\mathrm{Nu} = \Theta_1'(0) = \frac{\dot{q}_w}{k(T_c - T_w)\sqrt{c/\nu}} \tag{39}$$

The corresponding dimensional quantities can also be written as

$$(\tau_{zx})_w = \mu c \xi \cdot f''(0), \quad (\tau_{zy})_w = \mu c \zeta \cdot g''(0)$$

$$\text{and} \quad \dot{q}_w = k(T_c - T_w)\sqrt{c/\nu} \cdot \Theta_1'(0) \tag{40}$$

It was stated earlier that according to the assumptions made for the properties of the magnetic fluid and the flow configuration, a plausible value for the dimensionless magnetic interaction parameter β that saturates the fluid is $\beta = 1.5 \times 10^5$. It is worth examining, however, the behavior of the above mentioned dimensionless quantities $f''(0)$, $g''(0)$, and $\Theta_1'(0)$ supposing that the value of the magnetic interaction parameter β that saturates the magnetic fluid under consideration can vary from 1.3×10^5 to 1.5×10^5.

So, Fig. 8 shows the variation in the dimensionless skin friction coefficient $C_{f_\xi} = f''(0)$ against the magnetic interaction parameter β ($\beta^* = \beta \times 10^{-5}$). It is observed from this figure that $f''(0)$ increases as β increases, and this is in agreement with the variation in the velocity profile presented in Fig. 2. On the contrary, as it was expected from the analysis of the velocity field, the skin friction coefficient $g''(0)$ decreases with the magnetic interaction parameter β (Fig. 9). Finally, from Fig. 10, it is concluded that the amount of heat, per unit area and time $\dot{q}_w$ (or the dimensionless wall heat transfer parameter $\Theta_1'(0)$), flowing from the magnetic fluid to the elastic surface, that is, in the negative z-direction, decreases as the magnetic interaction parameter increases. This conclusion is in agreement with the results obtained from Fig. 5(b), showing the variation in the dimensional temperature $T(z)$ against the normal distance z from the elastic surface.

6 Conclusions

In the ferromagnetic case the velocity component $f'(\eta)$ is greater than the corresponding one in the hydrodynamic case. The opposite is true for the velocity component $g'(\eta)$. This fact is due to the influence of the Kelvin force on the flow field that is produced by the interaction between the motion of the fluid, which is induced by the stretching surface, and by the action of the magnetic field.

In the ferromagnetic case and very close to the elastic sheet, the velocity component in the y-direction takes negative values, which means that in that region the flow is reversed. The numerical analysis and the obtained numerical results showed that the dimensionless temperature $\Theta(\xi, \zeta, \eta)$ of the fluid is represented only by the dimensionless temperature $\Theta_1(\eta)$, and the dimensional temperature of the fluid is greater in the hydrodynamic case than the corresponding one in the ferromagnetic case. On the other hand, the dimensionless pressure $P(\xi, \zeta, \eta)$ is represented only by the function $P_1(\eta)$ in the hydrodynamic case and by the functions $P_1(\eta)$ and $P_3(\eta)$ in the ferromagnetic one. Finally, the magnetostatic pressure inside the boundary layer and the shearing stresses and the rate of heat transfer, on the stretching elastic surface, are noticeably affected by the variations in the magnetic interaction parameter β.

Nomenclature

a	=	thermal diffusivity, $m^2\ s^{-1}$
α	=	dimensionless distance
B	=	magnetic induction, Wb/m^2
B_s	=	saturation magnetic induction, Wb/m^2
c_1, c_2, c	=	dimensional constants, s^{-1}
c_p	=	specific heat at constant pressure, $J\ kg^{-1}\ K^{-1}$
d	=	distance of the wire from the x-axis, m
$\boldsymbol{\nabla}$	=	$(\partial/\partial x, \partial/\partial y, \partial/\partial z)$
∇^2	=	$(\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2)$ (Laplacian operator)
f', g'	=	dimensionless velocity components in the x- and y-directions
H	=	magnetic field intensity, $A\ m^{-1}$
I	=	electric current intensity, A
K	=	pyromagnetic coefficient, K^{-1}
k	=	thermal conductivity, $J\ s^{-1}\ m^{-1}\ K^{-1}$
M	=	fluid magnetization, $A\ m^{-1}$
M_s	=	saturation magnetization, $A\ m^{-1}$
P	=	dimensionless (magnetic and static) pressure
$P_\infty = p_\infty / c\mu$	=	dimensionless pressure far away from the sheet
Pr	=	Prandtl number
p	=	fluid pressure (magnetic and static), $N\ m^{-2}$
p_∞	=	fluid pressure far away from the sheet, $N\ m^{-2}$
$\boldsymbol{q}$	=	(u, v, w), velocity field
T	=	fluid temperature, K
T_c	=	Curie temperature (fluid temperature far away from the sheet), K
T_w	=	stretching sheet temperature, K
x, y, z	=	Cartesian coordinates, m
u, v, w	=	velocity components in the x, y, z-direction, $m\ s^{-1}$

Greek

β	=	dimensionless magnetic interaction parameter
δ_1, δ_2	=	dimensionless constants
ε	=	dimensionless temperature parameter
ξ, ζ, η	=	dimensionless coordinates
Θ	=	dimensionless temperature
λ	=	viscous dissipation parameter (dimensionless)
μ	=	dynamic viscosity, $kg\ m^{-1}\ s^{-1}$
μ_0	=	magnetic fluid permeability, $N\ A^{-2}$
ν	=	kinematical viscosity, $m^2\ s^{-1}$
π	=	$3.14159\cdots$
ρ	=	fluid density, $kg\ m^{-3}$
Φ	=	dissipation function, s^{-2}

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