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Numerical study of blood flow and heat transfer through stretching cylinder in the presence of a magnetic dipole

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In the present paper we study numerically the Biomagnetic Fluid Flow (BFD) (blood) and heat transfer through a stretching cylinder. The physical problem is formulated by a BFD model which incorporates both principles of FerroHydroDynamics (FHD) and MagnetoHydroDynamics (MHD). Thus, blood is considered to be an electrically conducting fluid which simultaneously exhibits polarization. The governing equations are non-dimensionalized using suitable similarity transformations and the resulting coupled non linear system of ordinary differential equations are solved using an efficient numerical technique which is based on a common finite differences method with central differencing, a tridiagonal matrix manipulation and an iterative procedure. Comparisons of our results with existed studies are made for some limiting case of the present study and found to be in a good agreement. The results are presented graphically for different values of the parameters with emphasis to the examination of FHD and MHD effect on the flow field as well as other physical quantities of interest, like skin friction coefficient and heat transfer on the wall.

KEYWORDS
Blood flow, FHD, Finite difference method, MHD, stretching cylinder

MSC (2010)
76D05, 76W05, 76Z99, 74F15

1 | INTRODUCTION

Biomagnetic fluid dynamics (BFD) has emerged as a new area of research in the study of the dynamics of biological/physiological fluids under the action of a magnetic field. Recently biomagnetic fluid research area is exponentially increasing because it is directed towards finding and developing the solutions to some of the human body related diseases and disorders such as designing artificial organs, creating nano-robots for surgery and developing advanced imaging and signal processing techniques for cancer, tumor and other life-threatening diseases. These research areas can have a direct real-world impact, for example, in the field of medical imaging based diagnostics (MRI, CT scan, ultrasound etc), and targeted transport of drugs or hyperthermia treatments [1–6].

Study of bio-magnetic fluid dynamics (BFD) that involves the influence of a magnetic field, we need to account for the principles of both ferrohydrodynamics (FHD) and magnetohydrodynamics (MHD). In FHD, we consider flows of electrically non-conducting magnetic fluids and assume that the flow is influenced by fluid magnetization that takes place due to the presence of a magnetic field. On the oteher hand, in MHD, we study the flow behavior of electrically conducting fluids, by ignoring the effect of polarization or magnetization.

Based on the principles of Ferrohydrodynamics (FHD) [7], Haik et al. [8] first proposed a BFD model, according to which the biomagnetic fluid is a Newtonian and electrically non-conducting magnetic fluid. They reported that the flow is appreciably affected by the magnetization of the fluid under the influence of high gradient magnetic fields. This BFD model was further extended by Tzirtzilakis [9] by incorporating both principles of FHD and MHD. The extended BFD model takes into account both magnetization and electrical conductivity of blood and the arising forces considered are the polarization and the Lorentz force. Several studies on biomagnetic fluid flows under different conditions were investigated by many researchers. Tzirtzilakis and Kafoussias [10] studied the flow of a heated ferrofluid over a linearly stretching sheet under the action of a magnetic field generated due to the presence of a magnetic dipole. The effect of electrical conductivity and magnetization on the BFD flow over a stretching sheet was studied by Murtaza et al. [11]. They found that the effect on the flow due to FHD is equally significant with that of MHD and consequently for this physical problem neither the electrical conductivity nor the magnetization of blood can be considered negligible. Flows of biomagnetic viscoelastic fluids in different situations were investigated theoretically by Misra and Shit [12–13]. These studies reveal that the presence of external magnetic field bears the potential of influencing the flow behaviour of biomagnetic viscoelastic fluids quite appreciably. An extensive work of biomagnetic fluid flows in various flow geometries has been done and some representative works are those of Tzirtzilakis [14–16] Misra and Sinha [17] and Tzirtzilakis and Xenos [18] and Tzirtzilakis and Tanoudis [19].

The MHD is related to the mutual interaction of fluid flow and magnetic fields. Abel et al. [20] and Lawrencer and Nageswara [21] studied the MHD viscoelastic fluid past a stretching sheet. Abd-Elaziz and Othman [22] investigated the effect of the Thomson heating and the Fourier's heat conduction, in the presence of a magnetic field. The MHD Boundary layer flow and heat transfer over a stretching cylinder is also an important problem due to its wide applications. Such analysis is used to blood flow measurements. Lin and Shih [23, 24] investigated laminar boundary layer heat transfer along horizontal and vertical moving cylinders with constant velocity and they found similarity solutions due to the curvature effect of the cylinder. Wang [25] investigated the fluid flow due to the stretching cylinder and found an exact similarity solution. Ishak et al. [26] investigated the incompressible flow of Newtonian fluid over stretching cylinder under the influence of magnetohydrodynamic effects. The solution was calculated numerically by applying the Keller-Box method. They illustrated that MHD causes decline in velocity profile while enhances the skin friction coefficient. Further Ishak and Nazar [27] reported that similarity solutions could be obtained by considering the cylinder stretched with a linear velocity in the axial direction. Ishak et al. [28] analysed the flow and heat transfer due to a stretching cylinder in the presence of magnetic field. They found that skin friction coefficient increases with magnetic parameter. Vajravelu et al. [29] investigated the axisymmetric magneto-hydrodynamic (MHD) flow and heat transfer at a non-isothermal stretching cylinder. They analyzed the effects of transverse curvature and the temperature dependent thermal conductivity on the magneto-hydrodynamic (MHD) axisymmetric flow and heat transfer characteristics and reported that impact of curvature parameter has to increase the horizontal velocity and the temperature fields. They also found that an increase in the value of magnetic parameter leads to a decrease in the velocity boundary layer thickness. Mukhopadhyay [30] examined the MHD boundary layer flow of viscous fluid over a stretching cylinder and analyzed that by increasing the curvature of the cylinder the rate of transport reduces significantly. Qasim et al. [31] discussed the magnetohydrodynamic (MHD) flow of ferrofluid along a stretching cylinder. They reported that surface shear stress and the heat transfer rate at the surface increase as the curvature parameter increases, i.e curvature helps to enhance the heat transfer. Hayet et al. [32] studied the axisymmetric flow of third grade fluid by a stretching cylinder in the presence of magnetic field and they concluded that the velocity and momentum boundary layer thickness are increasing funcvtions of curvature parameter. Nadeem et al. [33] discussed the partial slip conditions and MHD flow on an oblique stagnation point flow of rheological fluid. One of their findings was that the increment of the magnetic field leads to decrement of the the velocities. They also found that magnetic field has opposite behavior for both tangential and normal skin friction coefficients. Khan et al. [34] investigated the MHD boundary flow over a nonlinear stretching cylinder and they concluded that the velocity of the fluid particle decrease and drug force enhance with the magnetic parameter.

In view of all the above mentioned literature survey, it is observed that flow and heat transfer of biomagnetic fluids past a stretching cylinder is not investigated widely yet. To fill this gap, the main focus of the present analysis is to investigate the analysis of biomagnetic fluid (blood) flow and heat transfer through stretching cylinder with the variation of the magnetic interaction parameter. The model used take into account both magnetization and electrically conductivity arising in the magnetic fluid. The governing nonlinear Navier–Stokes equations in cylindrical polar coordinates are introduced. Similarity transformations are employed to render the nonlinear dimensional partial differential boundary layer equations into a set of ordinary differential equations. This set of non-linear differential equations is solved by using an efficient numerical technique which is based on the common finite differences method with central differencing, a tridiagonal matrix manipulation and an iterative procedure which is described by Kafoussias and Williams [35]. The influence of various parameters on velocity and temperature as well as skin friction coefficient and wall heat transfer are examined and discussed. The analysis

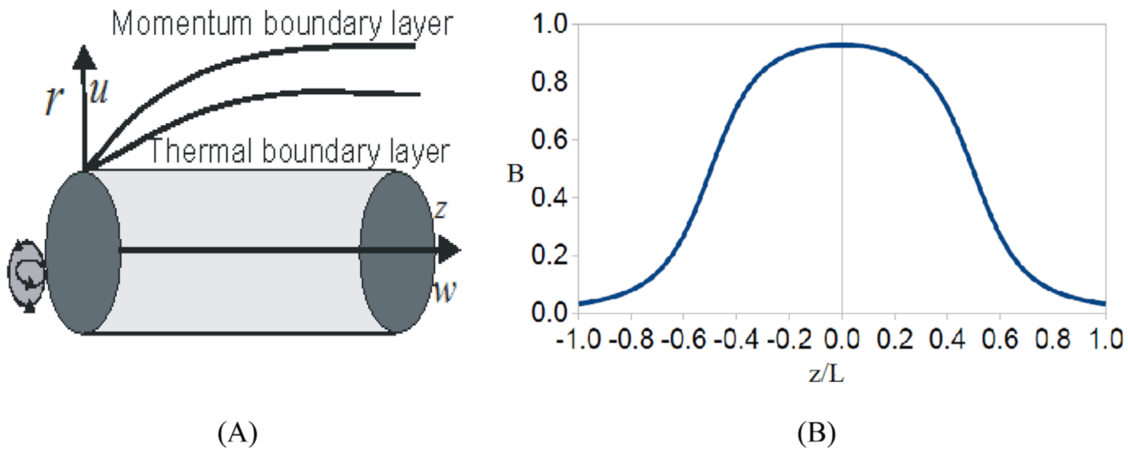

FIGURE 1 (A): Flow model and coordinate system, (B): Dimensionless applied magnetic field flux B

of the obtained results shows that the flow field is influenced appreciably by the magnetohydrodynamic and ferromagnetic parameters.

2 | MATHEMATICAL FORMULATION

Let us consider blood flow along the stretching cylinder with a constant radius a, whose physical model and geometric configuration of the flow is shown in Figure 1. Here r-axis is the radial direction and z axis is measured along the axis of the cylinder. Flow direction of the fluid is parallel to its axial direction. Based on the principles of FHD and MHD, Tzirtzilakis [9] proposed the blood is incompressible, viscous, laminar and electrically conducting fluid. A magnetic dipole is located parallel to the axial direction which generates a magnetic field of strength B_0 acting transverse to the flow direction. It is assumed that the surface of the cylinder is at constant temperature T_w and ambient fluid temperature is T_c, where $T_w < T_c$.

Under the above assumptions the equation of the continuity, momentum and energy equations are

Continuity equation

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

Momentum equations

$$u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial r^2} + \frac{\partial^2 w}{\partial z^2}\right) + \frac{\mu_0}{\rho}M\frac{\partial H}{\partial z} - \frac{\sigma}{\rho}B_0^2 w \tag{2}$$

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2}\right) \tag{3}$$

Energy equation

$$\rho C_p\left(u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}\right) + \mu_0 T\frac{\partial M}{\partial T}\left(u\frac{\partial H}{\partial r} + w\frac{\partial H}{\partial z}\right) = k\left(\frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{\sigma}{\rho}B_0 w^2 \tag{4}$$

where u and w are the velocity components measured along r and z axis, respectively. Also, ν, ρ, σ, k, μ_0, B_0 denotes the kinematic viscosity, density, electrical conductivity, thermal conductivity, magnetic permeability and transverse magnetic field respectively. The corresponding boundary conditions are

$$\begin{aligned} &w = u_w = \frac{u_0 z}{l}, \quad u = 0, \quad T = T_w \quad at \;\; r = a \\ &w \to 0, \quad T \to T_c \quad at \;\; r \to \infty \end{aligned} \tag{5}$$

Here $w = \frac{u_0 z}{l}$ is the stretching velocity, $u_0 > 0$ is the stretching rate and l is the reference length.

3 | MAGNETIC FIELD EQUATION

Now we find out the magnetic field strength intensity H of the cylinder whose radius is a, length l and is uniformly magnetized along its axis with $\boldsymbol{M} = M_o \boldsymbol{i}_z$.

The magnetic field intensity in terms of the scalar magnetic potential φ_M is introduced by

$$H = -\nabla \varphi_M$$

From the flux continuity law, magnetic scalar potential φ_M satisfies Poisson's equation.

$$\nabla^2 \varphi_M = -\rho_M \text{ where } \rho_M = \nabla . \boldsymbol{M}$$

where ρ_m is the magnetic charge density.

Because the magnetization is constant inside the cylinder, its divergence is zero there. The magnetic charge density must reside on the surface. Drawing a closed surface around the surface charge density and shrinking it down leads to $(\boldsymbol{M}_2 - \boldsymbol{M}_1).n = -\sigma_M$. Outside there is no magnetization $\boldsymbol{M}_2 = 0$ and inside we know $\boldsymbol{M}_1 = \boldsymbol{M}_0 \bar{z}$ so that we have:

$$\sigma_M = \pm \boldsymbol{M}_0 \bar{z}.n$$

The positive and negative signs refer to the upper and lower surfaces.

This is best summed up as:

$$\rho_M = M_0(\delta(z - l/2) - \delta(z + l/2)) \text{ for } \rho < a \text{ and } \rho_M = 0 \text{ for } \rho > a$$

The Poisson equation for the magnetic scalar potential has the general solution:

$$\varphi_M = \frac{1}{4\pi} \int_V \frac{\rho_M(r')}{r - r'} dv$$

along the axis of cylinder, these integrals become

$$\varphi_M = \frac{M_0}{2} \left[\sqrt{a^2 + \left((z - l/2)^2\right)} - \sqrt{a^2 + \left((z + l/2)^2\right)} - |z - l/2| + |z + l/2| \right]$$

$$H = -\nabla \varphi_M = -\frac{M_0}{2} \left[\frac{z - l/2}{\sqrt{a^2 + (z - l/2)^2}} - \frac{z + l/2}{\sqrt{a^2 + (z + l/2)^2}} - \frac{z - l/2}{|z - l/2|} - \frac{z + l/2}{|z + l/2|} \right]$$

For $-l/2 < z < l/2$, the magnetic field strength intensity is

$$H_z = -\frac{M_0}{2} \left[\frac{z - l/2}{\sqrt{a^2 + (z - l/2)^2}} - \frac{z + l/2}{\sqrt{a^2 + (z + l/2)^2}} + 2 \right]$$

and the gradient of H is given by

$$\frac{\partial H}{\partial z} = -\frac{M_0}{2} \frac{3zl}{a^3} \tag{6}$$

4 | TRANSFORMATION OF EQUATIONS

The stream function ψ is defined as $u = -\frac{1}{r}\frac{\partial \psi}{\partial z}$, $w = \frac{1}{r}\frac{\partial \psi}{\partial r}$

Following Vajravelu et al. [29], we introduce the similarity variables

$$\eta = \frac{r^2 - a^2}{2a}\sqrt{\frac{u_w}{\nu l}}, \quad \psi = a\sqrt{u_w \nu z} f, \quad \theta(\xi,\eta) = \frac{T_c - T}{T_c - T_w} = \theta_1(\eta) + z^2\theta_2(\eta),$$

$$P(\xi,\eta) = \frac{u_0}{l}\mu\left(P_1 + z^2 P_2\right), \quad M = K(T_c - T) \tag{7}$$

and the dimensionless velocity

$$u = -\frac{1}{r}\frac{\partial\psi}{\partial z} = -\frac{a}{r}\sqrt{\frac{u_0\nu}{l}} f, \quad w = \frac{1}{r}\frac{\partial\psi}{\partial r} = \frac{u_0\nu}{l} f' \tag{8}$$

equation (1) is automatically satisfied and using Equations (6)–(8) in (2), (3) and (4) we get the following set of nonlinear ordinary differential equations

$$(1 + 2\eta\gamma)f''' + (2\gamma + f)f'' - (f' + M)f' - \beta\theta_1 - \frac{2\mu}{\rho^2 u_0}P_2 = 0 \tag{9}$$

$$(1 + 2\eta\gamma)^2 {P_1}' - (1 + 2\eta\gamma)^2 f'' + (1 + 2\eta\gamma) f f' - \gamma f^2 = 0 \tag{10}$$

$$(1 + 2\eta\gamma)^2 {P_2}' = 0 \tag{11}$$

$$(1 + 2\eta\gamma){\theta_1}'' + (P_r f + \gamma)f'' - (f' + M){\theta_1}' + 2a^2\gamma^2\theta_2 = 0 \tag{12}$$

$$(1 + 2\eta\gamma)\theta_2'' + P_r(f\theta_2' - 2f'\theta_2) + \gamma\theta_1' + \beta\lambda(\varepsilon - \theta_2)f' - M\lambda f'^2 = 0 \tag{13}$$

With corresponding associated boundary conditions:

$$f' = 1,\ \theta_1 = 1,\ \theta_2 = 0 \quad at \ \ \eta = 0$$

$$f' \to 0,\ \theta_1 \to 0,\ \theta_2 \to 0,\ P_1 \to 0 \quad at \ \ \eta \to \infty \tag{14}$$

where $\beta = \frac{3}{2}[\mu_0 M_0 K(T_c - T_w)l^3/\rho a^3 {u_0}^2]$ the ferromagnetic interaction parameter appearing due to the principles of FHD, $M = \sigma {B_0}^2 l/\rho u_0$ the magnetic parameter appearing in MHD which is the square of the Hartman number, $\lambda = (\frac{{u_0}^2}{l})\mu/K(T_c - T_w)$ the viscous dissipation parameter, $\gamma = \sqrt{\nu l/a^2 u_0}$ the curvature parameter, $\Pr = \mu c_p/k$ the Prandtl number, $\varepsilon = T_c/(T_c - T_w)$ the temperature number.

We note here that for $\beta = M = 0$ we have the pure hydrodynamic problem of the stretching cylinder. Moreover, when $\beta = 0$ and $M \neq 0$ the problem is the MHD stretching cylinder flow whereas, for $\beta \neq 0$ and $M = 0$ the physical problem is reduced to the FHD stretching cylinder flow.

5 | NUMERICAL METHOD

The system of Equations (10)-(13) subject to the boundary conditions (14) is solved numerically using a numerical technique described in detail in Kafoussias and Williams [35]. This technique attain solution of two point boundary value problems governed by second order non-linear ordinary differential equations. It has good stability characteristics, is simple accurate and efficient and constitutes of the following essential features:

(i) it is based on the common finite differences method with central differencing
(ii) on a tridiagonal matrix manipulation and finally
(iii) on an iterative procedure.

In this method, it is essential to select a suitable finite value of n_∞. The step size $h = 0.01$ issued to obtain the numerical solution with n_∞ and and appropriate n_∞ values as $(y \to \infty)$ must be determined. The different initial guesses were made taking into account the convergence. The process is repeated until the results are corrected up to a desired accuracy. By trial and error, we get $n_\infty = 1$ and the tolerance between the iterations is set at $\varepsilon = 10^{-4}$. In this model we also performed calculations for

TABLE 1 Numerical values of $(-f''(0))$ with various values of M and γ are compared with the result obtained by Vajravelue et al. [29]

M	γ	Vajravelue et al. [29]	Present result
0.5	$\gamma = 0.0$	1.224745	1.224682
0.5	$\gamma = 0.25$	1.328505	1.328422
0.5	$\gamma = 0.5$	1.427151	1.427204
0.5	$\gamma = 1.0$	1.613858	1.618042
1.0	$\gamma = 0.0$	1.414214	1.414115
1.0	$\gamma = 0.25$	1.523163	1.522972
1.0	$\gamma = 0.5$	1.626496	1.626203
1.0	$\gamma = 1.0$	1.821302	1.820895

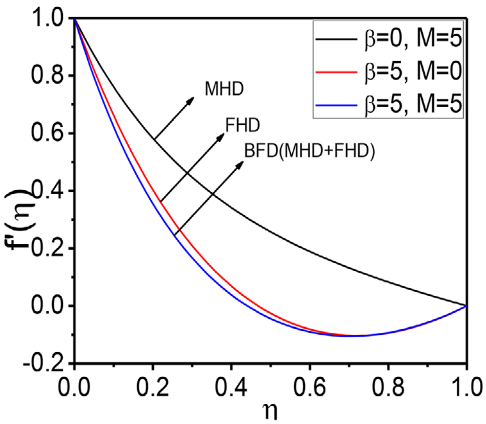

FIGURE 2 Velocity profile for FHD, MHD and BFD

$h = \Delta\eta = 0.001$ and convergence criterion at $\varepsilon = 10^{-5}$ and no significant differences were found. The numerical methodology used is also briefly discussed in Murtaza et al. [11].

6 | RESULTS AND DISCUSSIONS

In this section, the influence of various parameters on velocity, pressure, temperature, skin friction and heat transfer rate of blood are discussed. First, realistic values should be assigned at the parameters entering the physical problem. To attain this, we use values previously documented in the relative literature. Since the fluid is assumed to be blood so we assign: $\rho = 1050\ \mathrm{kg/m^3}$, $\mu = 3.2 \times 10^{-3}\ \mathrm{kgm^{-1}s^{-1}}$[10], $\sigma = 0.8\ \mathrm{sm^{-1}}$, $C_p = 14.65\ \mathrm{JKg^{-1}.K^{-1}}$, $k = 2.2 \times 10^{-3}\ \mathrm{Jm^{-1}s^{-1}k^{-1}}$ [10, 14, 18] and hence $P_r = \frac{\mu C_p}{k} = 21$, for a human body temperature [19] $T_w = 37°\mathrm{C}$ whereas the body curie temperature is $T_c = 41°\mathrm{C}$, hence the dimensionless temperature is $\varepsilon = 78.5$. The present numerical investigation has been carried out with the ferromagnetic interaction parameter B = 0, 5, 8, 10, 41 [10, 11, 14, 19] and magnetohydrodynamic interaction parameter M = 0, 5, 10 [19], $\gamma = 0.5,\ 1,\ 1.5$ [29]. Noted that $M = 0,\ \beta \neq 0$ corresponds to pure ferrohydrodynamic flow, $M \neq 0,\ \beta = 0$ corresponds to pure magnetohydrodynamic flow and $M \neq 0,\ \beta \neq 0$ corresponds to mixed ferrohydrodynamic and magnetohydrodynamic flow i.e. biomagnetic fluid flow modelled by the extended BFD model [9].

For the validation of the numerical results, for some limited case, when the ferromagnetic parameter is absent i.e $M \neq 0,\ \beta = 0$, the present numerical results are compared with the results of Vajravelu et al. [29] for $-f''(0)$. The comparisons of $-f''(0)$ for various values of M and γ, are given in Table 1 and found to be in good agreement with those calculated in the present paper.

Figures (2) to (6) demonstrate the influence of ferromagnetic, magnetohydrodynamic and curvature parameter on the axial and transverse velocity, respectively. From Figure 2 it is apparent that the decrement of the axial velocity f' is greater for pure ferrohydrodynamic flow than pure MHD one. This decrement is more effective for the BFD flow. It is clear that due to the presence of a magnetic field both Kelvin and Lorentz forces are acting against the flow. This resistive force slows down the fluid velocity component i.e. boundary layer thickness decreases. These findings are consistent with previous documented findings [10, 11, 19]. Moreover, from Figure 3 we observe that the flow can be reversed for various value of the MHD parameter. The point of the detachment of the flow from the wall (f'), cross flow is shown far away from the sheet for smaller value of FHD

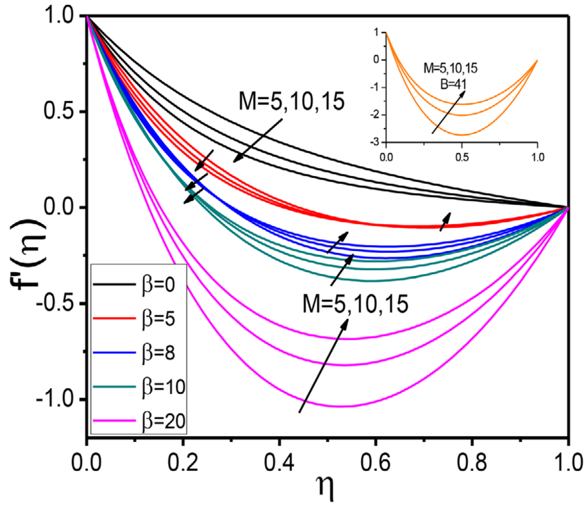

FIGURE 3 Axial velocity profile for various values of M and β

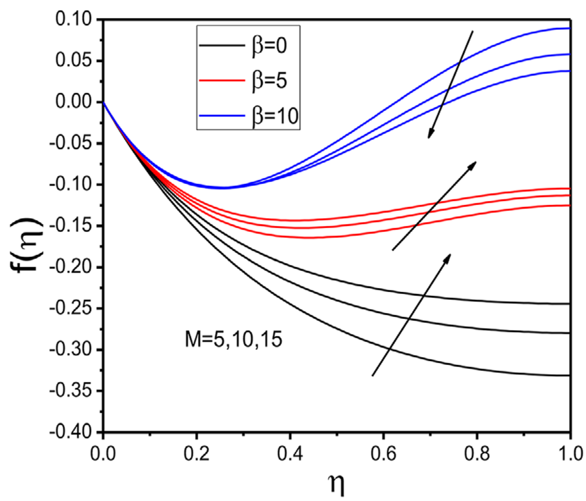

FIGURE 4 Transverse velocity profile for various values of M and β

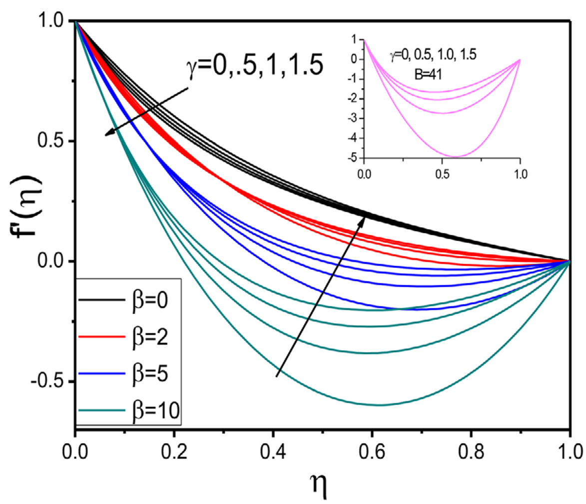

FIGURE 5 Axial Velocity profile for various values of γ and β

parameter and for increasing value of ferromagnetic parameter cross flow is shown near the sheet and finally for larger value of FHD parameter, we observe that there is no cross flow (Figure 3).

In Figure 4 shows the variation of the transverse velocity for different values of ferromagnetic, and magnetohydrodynamic parameters. From Figure 4 we generally observe that, all the profiles of the transverse velocity are generally increased as M increases for low values of β and the opposite is happening for $\beta = 10$. Moreover, for the lower values of β i.e. $\beta < 5$ and for a specific value of β, increment of M leads to increment of the transverse velocity. This is due to the fact that variation of B

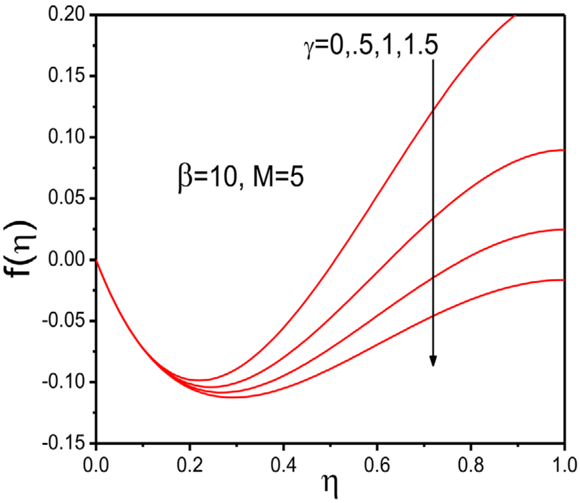

FIGURE 6 Transverse velocity profile for various values of γ and β

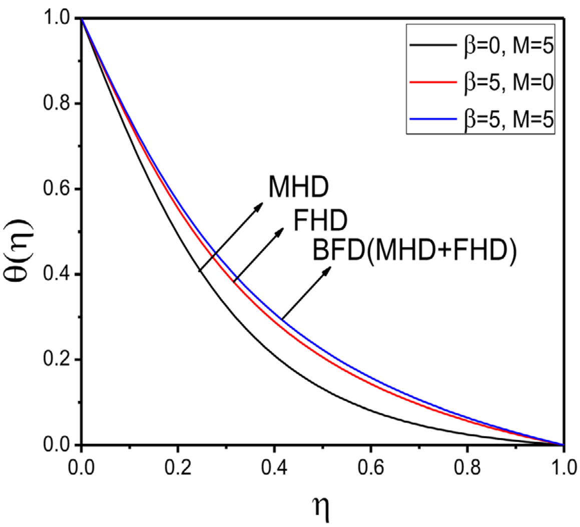

FIGURE 7 Temperature profile for FHD, MHD and BFD

leads to the variation of the Lorentz force due to magnetic field, and the Lorentz force produces more resistance to transport phenomena. It is noted that for larger values of ferromagnetic parameter (say $\beta = 10$), the transverse velocity shows the opposite behavior with increasing magnetohydrodynamic parameter.

Figures 5 and 6 show the variation of the transverse and axial velocity for different values of ferromagneic, magnetohydrodynamic. and curvature parameter. We notice from these figures that the transverse velocity profile decreases but the axial velocity profile increases with increasing values of the curvature parameter (Figures 5 and 6). The effect of increasing values of the curvature parameter is to increase the axial velocity and thereby enhance the boundary layer thickness. That is, the boundary layer thickness is greater for higher values of the curvature parameter. From Figure 5 we also observe that for different values of curvature parameter the cross flow has produced. This cross flow is near the sheet for larger value of ferromagnetic parameter and after certain values of β cross flow is vanished.

Figure 7 shows the temperature profile for different cases of MHD, FHD and extended BFD flows. We observe from this figure that the temperature is increased in all cases. The most effective increment is attained for the extended BFD cases.

Figures 8 and 9 illustrate the variation of the temperature distributions with ferromagnetic, magnetohydrodynamic and curvature parameter. We observe that, for valuew of β less than approximately 8, the temperature distribution is increased as M increases. For greater values of β the temperature is reduced with the increment of M. Analogous behavior is observed for the effect of the curvature along with the variation of β, pictured at Figure 9. Namely, for β less than approximately 8, increment of γ results to increment of $\theta(\eta)$ whereas, the opposite is true for value of β greater than 8.

Figures 10 and 11 illustrate the pressure distribution for various values of β, M and γ. It is noticed that the pressure control is more effective for extended BFD case than MHD or FHD. We also observe that the pressure distribution is decreased near the wall and reverse trend is shown far away from the sheet with the increment of the curvature parameter.

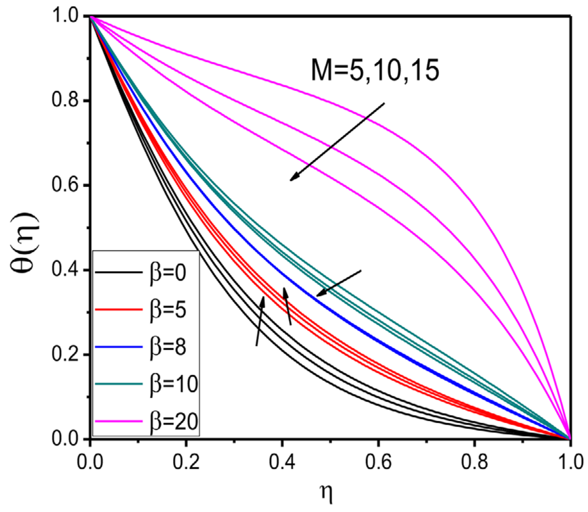

FIGURE 8 Temperature profile for various values of M and β

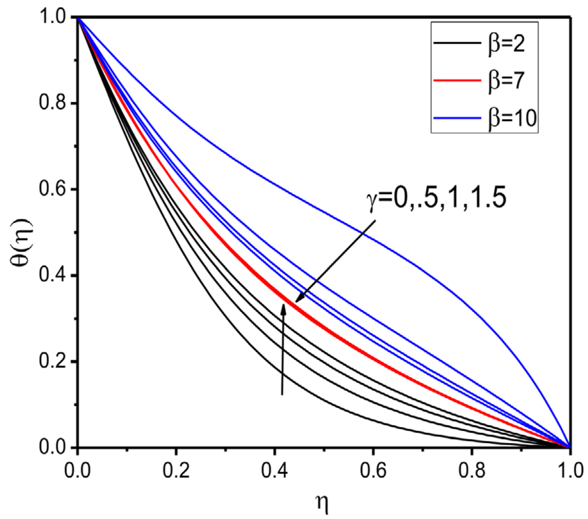

FIGURE 9 Temperature profile for various values of γ and β

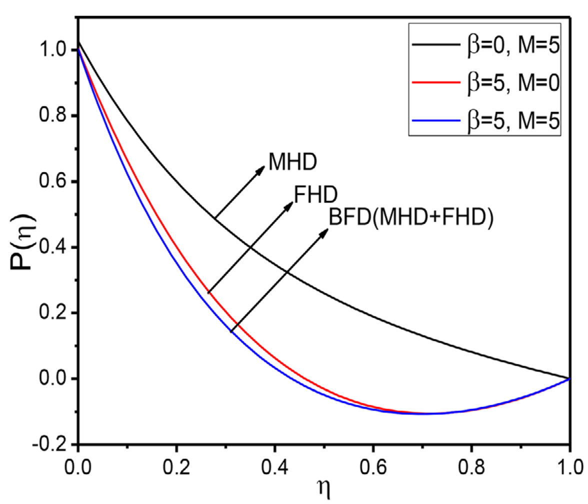

FIGURE 10 Pressure profile for FHD, MHD and BFD

Figures 12 to 15 represent the skin friction coefficient and wall heat transfer rate with M, β and γ. The variation of $-f''(0)$ is shown in Figures 12 and 13. Increment of γ leads to almost linear increment of $-f''(0)$ for smaller values of β and for larger value of β its increment is not linear and the same behavior is observed with the increment of M. From Figure 13 we observe the $-f''(0)$ is a linear increment with b and γ. It is seen that the values of $f''(0)$ are always negative. Physically, negative sign of $f''(0)$ implies that the stretching sheet exerts a drag force on the fluid that cause the movement of the fluid on the surface.

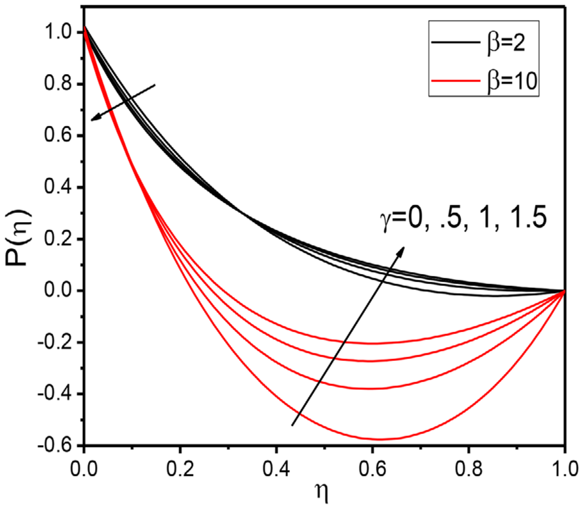

FIGURE 11 Pressure profile for various values of γ and β

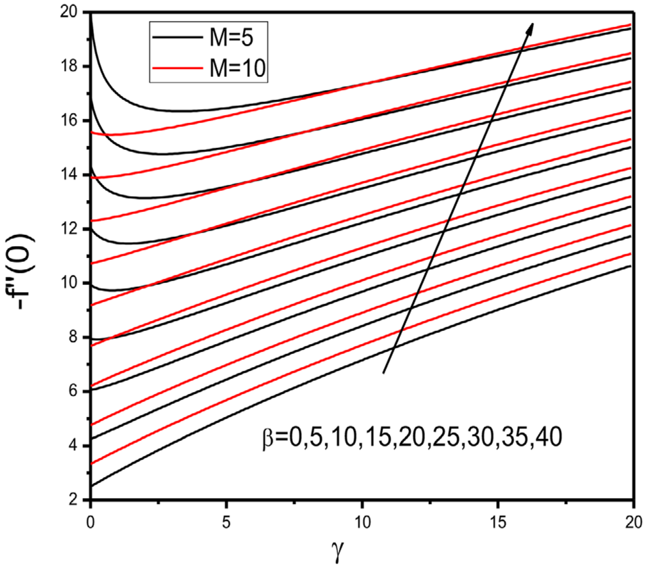

FIGURE 12 Skin friction coefficient with γ for various values of β and M

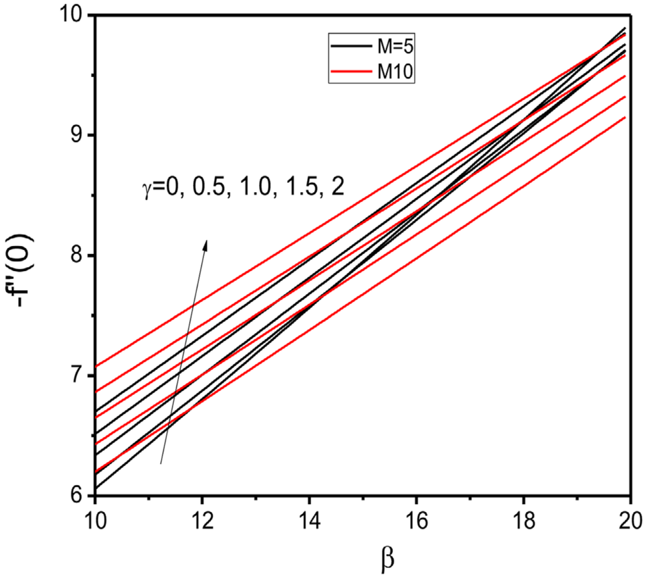

FIGURE 13 Skin friction coefficient with β for various values of γ and M

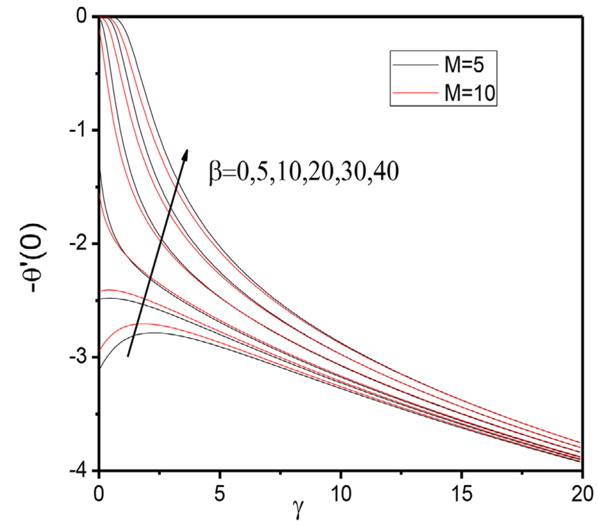

FIGURE 14 Rate of heat transfer with γ for various values of β and M

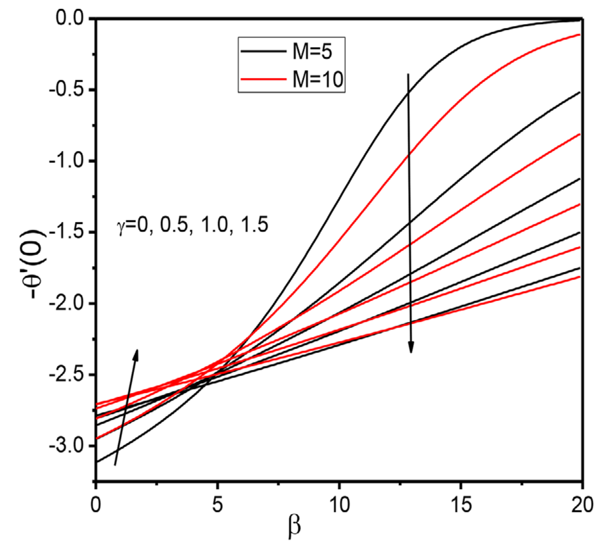

FIGURE 15 Rate of heat transfer with β for various values of γ and M

The variation of the wall heat transfer parameter $-\theta'(0)$ with β, M and γ is shown at Figures 14 and 15. For this figure we observe that $-\theta'(0)$ decrease with γ and its decrease is not linear. Here we observe that the major variation is observed with the variation of β rather than M (for a specific value of β). As β is increased the reduction of $-\theta'(0)$ with the increment of β is observed at Figure 15. This increment is greater with the increment of γ for relatively small values of β. The opposite is true for values of β greater than 5.

7 | CONCLUSION

The effect of magnetic field on two-dimensional blood flow and heat transfer through a cylindrical tube has been studied. The governing equations reduced to a set of ODE by using appropriate transformation and these equations are solved numerically using finite difference scheme based on central differencing and tridiagonal matrix. From the above investigation, we can we can draw the following conclusions:

(i) Blood flow control is appreciably reduced and temperature is enhanced for the extended BFD formulation rather than the formulation of FHD or MHD.

(ii) The effects of the magnetic number on the transverse velocity as well as on the temperature profiles are showing an interesting behavior. With the increase of FHD number due to the magnetization, the temperature as well as the transverse velocity are generally increased in the boundary layer. This increment is further enhanced with the increment of M for the lower values of β. However, for a specific $\beta > 5$ increment of M has the opposite effect and the transverse velocity and temperature distribution are reduced. Of course this reduction is not enough to counterbalance the increment caused by the increment of β.

(iii) The transverse velocity profile decreases near the wall and the reverse trends is shown far away from the wall with the increment of the curvature parameter for smaller values of the FHD parameter. On the other hand, forlarger values of FHD number ($\beta > 5$) the velocity is increased and the temperature is decreased with the increment of the curvature parameter.

(iv) The magnitude of the skin friction coefficient as a function of curvature parameter increases with the increment of the magnetic parameter whereas the rate of heat transfer is decreased.

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