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Vol. 26, No. 1, 2017
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Distributed worldwide by Springer. *Journal of Engineering Thermophysics* ISSN 1810-2328.

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ISSN 1810-2328, Journal of Engineering Thermophysics, 2017, Vol. 26, No. 1, pp. 1–11. © Pleiades Publishing, Ltd., 2017.

Effects of Radiation and Thermal Conductivity on MHD Boundary Layer Flow with Heat Transfer along a Vertical Stretching Sheet in a Porous Medium

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Received February 18, 2015

Abstract—A steady two-dimensional free convective flow of a viscous incompressible fluid along a vertical stretching sheet with the effect of magnetic field, radiation and variable thermal conductivity in porous media is analyzed. The nonlinear partial differential equations, governing the flow field under consideration, have been transformed by a similarity transformation into a system of nonlinear ordinary differential equations and then solved numerically. Resulting non-dimensional velocity and temperature profiles are then presented graphically for different values of the parameters. Finally, the effects of the pertinent parameters, which are of physical and engineering interest, are examined both in graphical and tabular form.

DOI: 10.1134/S1810232817010015

1. INTRODUCTION

Boundary layer flows and heat transfer over a linearly stretched surface have received considerable attention in recent years because of the various possible engineering and metallurgical applications such as hot rolling, wire drawing, metal and plastic extrusion, continuous casting and glass fibre production. Sakiadis [1] was the first to study a boundary layer flow over a stretched surface moving with a constant velocity. Erickson et al. [2] extended the work of Sakiadis and later Chen and Char [3], Elbashbeshy [4] investigated the effects of variable surface temperature and heat flux on the heat transfer characteristics of a linearly stretching sheet subject to blowing or suction.

The magneto-hydrodynamics (MHD) of an electrically conducting fluid is encountered in many problems in geophysics, astrophysics, engineering applications and other industrial areas. Focusing on moving and stretching surfaces, Kumar et al. [5] studied MHD flow and heat transfer on a continuously moving vertical plate and Ishak et al. [6] analyzed the same case over a stretching sheet.

At high operating temperature, the effect of radiation on MHD flow can be quite significant. Nuclear power plants, gas turbines, and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. Takhar et al. [7] studied the radiation effects on MHD free-convection flow of a gas past a semi-infinite vertical plate. Hossain and Takhar [8] determined the effect of radiation on mixed convection flow of an optically thick viscous incompressible flow past a heated vertical porous plate with a uniform surface temperature and a uniform rate of suction where radiation is included by assuming Rosseland diffusion approximation. Ghaly [9] considered radiation effect on a steady flow, whereas Raptis and Massalas [10] and El-Aziz [11] analyzed the unsteady cases. Shateyi et al. [12] studied magneto-hydrodynamic flow past a vertical plate with radiative heat transfer.

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Radiative heat transfer in porous media has also been investigated by many researchers; some of them are Raptis and Kafoussias [13], Sattar et al. [14], Kim [15], and Whitaker [16]. Performances of polymer extrusion processes can be improved by a thermally controlled environment, where the radiation effects may become important, are discussed by Mukhopadhyay [17] in study on mixed convection flow and heat transfer over a porous stretching surface in a porous medium.

Chaim [18] studied heat transfer in a fluid flow of low Prandtl number with variable thermal conductivity, induced due to stretching sheet, and compared the numerical results with perturbation solution. Recently, Sharma and Singh [19] investigated the MHD flow near a stagnation point with variable thermal conductivity and heat source/sink and Vyas and Rai [20] analyzed the case over a non-isothermal stretching sheet in a porous medium with radiation and variable thermal conductivity.

The aim of the present paper is to investigate effects of thermal radiation on a steady boundary layer flow with temperature-dependent thermal conductivity due to a stretching sheet through porous media in the presence of transverse magnetic field near a stagnation point. Linear stretching of the sheet is considered because of its simplicity in modeling of the flow and heat transfer over a stretching surface and further it permits the similarity solution, which is useful in understanding the interaction of flow field with temperature field.

2. MATHEMATICAL FORMULATION

Let us consider a steady two-dimensional MHD free convection laminar boundary layer flow of a viscous incompressible and electrically conducting fluid along a vertical stretching sheet with variable thermal conductivity under the influence of thermal radiation in a porous medium. Introducing the Cartesian coordinate system, the x axis is taken along the stretching sheet in the vertically upward direction and the y axis is taken as normal to the sheet. Two equal and opposite forces are introduced along the x axis, so that the sheet is linearly stretched keeping the origin fixed. The plate is maintained at a constant temperature T_w and the ambient temperature of the flow is T_∞. The fluid flows in parallel with velocity u_e over the sheet to facilitate the effect of heat-transfer process.

The fluid is considered to be gray, absorbing-emitting radiation but non-scattering medium. Assuming the fluid to be Newtonian, without phase change and optically dense, we further assume that both the fluid and the porous medium are in local thermal equilibrium. The Rosseland approximation is used to describe the radiative heat flux in the energy equation. The radioactive heat flux in the x direction is considered negligible in comparison to the y direction. A strong magnetic field is applied in the y direction. The electrical current flowing in the fluid gives rise to an induced magnetic field if the fluid were an electrical insulator, but here we have taken the fluid to be electrically conducting. Hence, only the applied magnetic field B_0 plays a role, which gives rise to magnetic forces $F_x = \frac{\sigma B_0^2 u}{\rho}$ in x direction, where σ is the electrical conductivity, B_0 is the uniform magnetic field strength, and ρ is the density of the fluid. Considering all the physical properties of the fluid constant except the thermal conductivity, which varies linearly with temperature, and neglecting the effect of viscous dissipation in the energy equation, the governing boundary layer equations are

Equation of continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{1}$$

Momentum equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \hat{v}\frac{\partial^2 u}{\partial y^2} + u_e\frac{du_e}{dx} - \left(\frac{v}{K} + \frac{\sigma B_0^2}{\rho}\right) u. \tag{2}$$

Energy equation:

$$\rho c_p \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) - \frac{\partial q_r}{\partial y}, \tag{3}$$

where u and v are the velocity components in the x and y directions, respectively, ν is the kinematic viscosity, $\hat{\upsilon}$ is the effective kinematic viscosity, K is the permeability, T is the temperature, k is the variable thermal conductivity of the fluid, c_p is the specific heat at constant pressure, and q_r is the radiative heat flux.

The boundary conditions are

$$\left.\begin{aligned} &u = bx,\ \ v = 0,\ \ T = T_w \quad \text{at}\ \ y = 0, \\ &u = u_e(x) = ax,\ \ T = T_\infty \ \ \text{as}\ \ y \to \infty. \end{aligned}\right\} \tag{4}$$

This choice of linear external velocity like the stretching velocity facilitates the similarity solutions of the problem and may be found also in other studies akin to the present one, such as Attia [21].

The radiative heat flux q_r is described by the Rosseland approximation (Brewster [22]) such that

$$q_r = -\frac{4\sigma_1}{3k_1}\frac{\partial T^4}{\partial y}, \tag{5}$$

where σ_1 is the Stefan–Boltzman constant and k_1 is the Rosseland mean absorption coefficient. It is assumed that the temperature differences within the flow are sufficiently small such that T^4 can be expressed as a linear function of temperature. This is accomplished by expanding T^4 in a Taylor series around the free stream temperature T_∞ and neglecting higher-order terms. This results in the following approximation:

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4. \tag{6}$$

Therefore, Eq. (3) on using Eq. (6) becomes

$$\rho\, c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{16\sigma_1 T_\infty^3}{3k_1}\frac{\partial^2 T}{\partial y^2}. \tag{7}$$

In order to obtain a similarity solution to the problem, we now introduce the following dimensionless variables:

$$\eta = y\left(\frac{b}{\upsilon}\right)^{1/2}, \quad \psi = \sqrt{b\upsilon}\,x f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \tag{8}$$

where ψ is the stream function, η is the dimensionless distance normal to the sheet, f is the dimensionless stream function, and θ is the dimensionless fluid temperature. Now

$$u = bx f'(\eta), \quad v = -(b\upsilon)^{1/2} f(\eta). \tag{9}$$

We also assume that the wall temperature is variable as a power-law function of x

$$T_w\,(x) = T_\infty + Dx^\alpha, \tag{10a}$$

where D and α are positive constants. Following Chaim [17], the thermal conductivity k is taken of the form

$$k = k_\infty(1 + \varepsilon\theta), \tag{10b}$$

where k_∞ is the fluid free stream conductivity and ε is a small parameter.

Using the transformations from Eqs. (8), (9) and (10) in Eqs. (2) and (7), we get the following dimensionless equations:

$$\Lambda f''' + f f'' - (f')^2 - R f' + \lambda^2 = 0, \tag{11}$$

$$(1+\varepsilon)\theta'' + \varepsilon \theta'^2 + \frac{4}{3} N \theta'' = \Pr(\alpha \theta f' - f \theta'), \tag{12}$$

where $\lambda = \frac{a}{b}$ is the ratio of free stream velocity to stretching sheet velocity, $R = \frac{\upsilon}{Kb} + \frac{\sigma B_0^2}{\rho b}$ is the combined effect of permeability and magnetic field, $\Lambda = \frac{\hat{\upsilon}}{\upsilon}$ is the viscosity ratio, α is the wall temperature index, ε is the perturbation parameter, $\Pr = \frac{\mu c_p}{k}$ is the Prandtl number, and $N = \frac{4\sigma_1 T_\infty^3}{k_1 k_\infty}$ is the radiation parameter.

The transformed boundary conditions are:

$$\left.\begin{aligned} &f = 0, \;\; f' = 1, \;\; \theta = 1 \;\; \text{at} \;\; \eta = 0, \\ &f' = \lambda, \;\; \theta = 0 \qquad\qquad \text{as} \;\; \eta \to \infty. \end{aligned}\right\} \tag{13}$$

The quantities of physical interest, namely, the local skin friction coefficient C_f and the rate of heat transfer in terms of local Nusselt number Nu are prescribed by

$$C_f = \frac{\tau_w}{(1/2)\,\rho\, u^2}, \quad Nu = \frac{q_w x}{k(T_w - T_\infty)}, \tag{14}$$

where $\tau_w = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0}$ is the skin friction and $q_w = -k \left(\frac{\partial T}{\partial y}\right)_{y=0}$ is the heat transfer from the sheet. Using these we obtain

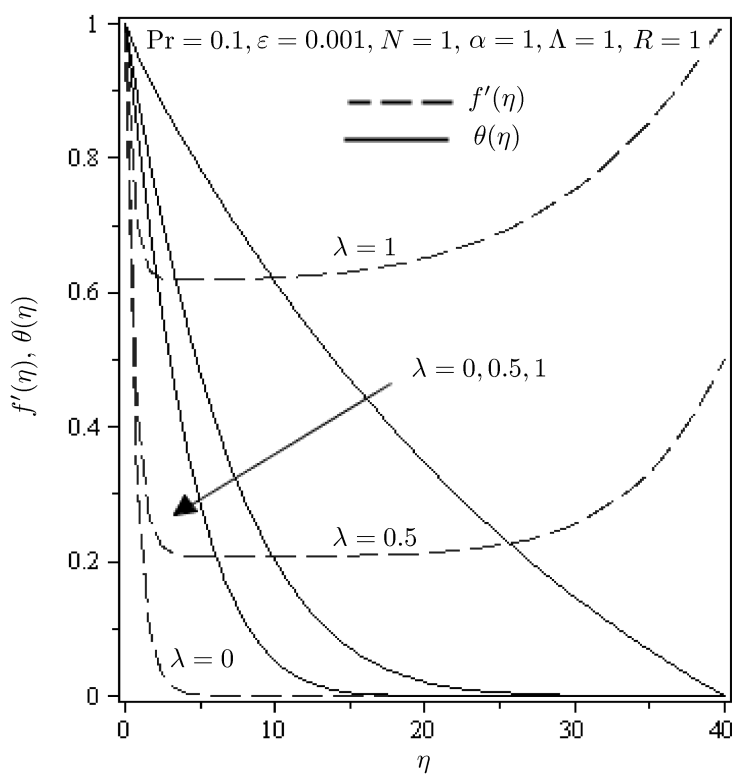

Fig. 1. Flow profiles for λ.

$$C_f \mathrm{Re}^{1/2} = f''(0), \quad Nu/\mathrm{Re}^{1/2} = -\theta'(0), \tag{15}$$

where $\mathrm{Re} = ux/\upsilon$ is the local Reynold's number.

3. RESULTS AND DISCUSSION

The system of Eqs. (11) and (12) subject to the boundary conditions (13) has been solved numerically for some values of the governing parameters using Maple.

Figure 1 shows the effect of the ratio of free stream velocity to stretching sheet velocity, λ on the velocity and temperature profiles. The thermal boundary layers decrease as λ is increased. However, the velocity profiles show different characteristics. For $\lambda = 0$ (meaning a fluid at rest far from the stretching sheet), the velocity boundary layer decreases very rapidly, but for $\lambda = 0.5$ the velocity profile decreases very sharply up to η reaches 2.5, then it remains constant at $f' = 0.2$ as η reaches 20.0 and after that it starts to increase and satisfies the boundary condition at $\eta = 40.0$. The same trend can be seen for $\lambda = 1.0$. But this time the momentum boundary layer remains constant at $f' = 0.62$ for $2.0 \leq \eta \leq 15.0$ and starts to rise to satisfy the boundary condition at $\eta = 40.0$.

Figure 2 illustrates the effect of viscosity ratio Λ on the boundary layers. As the viscosity ratio is increased the thermal boundary layers decrease. However, the momentum boundary layers show different characteristics. They start to fall very rapidly, then remain constant as η increases and then at a certain point around they start to rise. It should be noted here that the boundary layer thicknesses remain constant for $5.0 \leq \eta \leq 25.0$ and overall, the velocity profiles increase as the values of Λ go up/increase.

It is noticed the weak influence of the viscosity ratio Λ on the velocity and temperature profiles. Although there is yet demonstrated that in porous media this parameter may be quite different from unity, many authors take in their analysis $\Lambda = 1$, see Ziabahksh et al. [23] for a stretching sheet immersed in a porous medium.

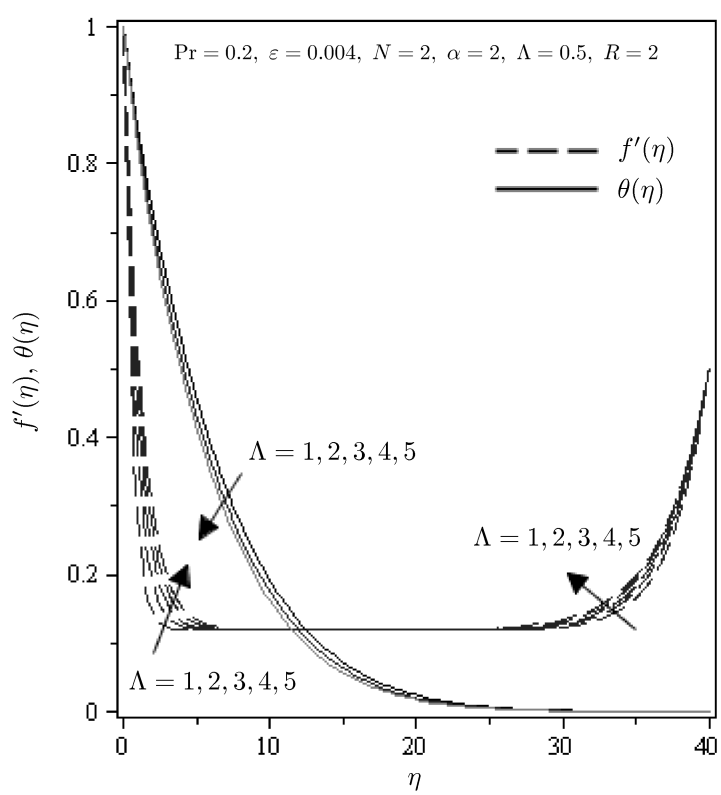

Fig. 2. Flow profiles for Λ.

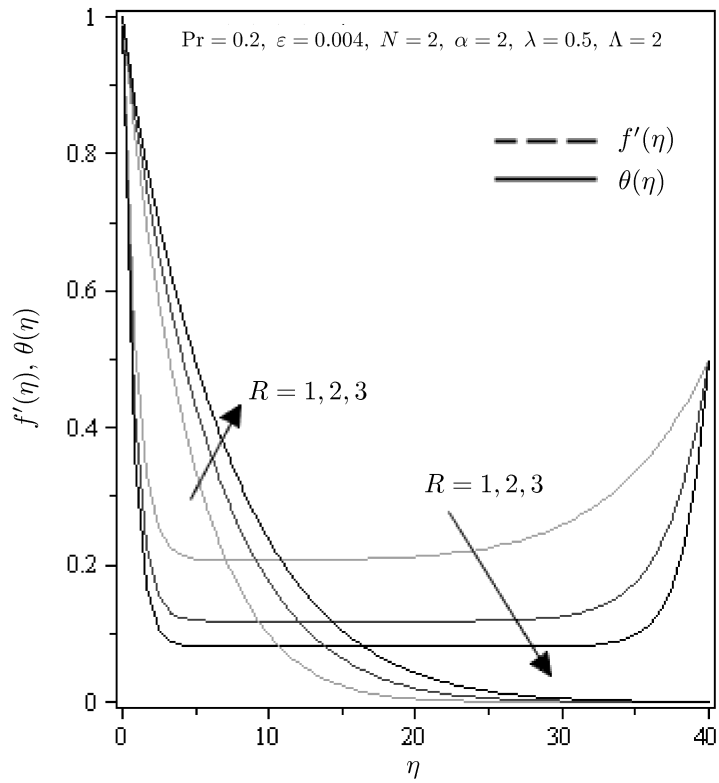

Fig. 3. Flow profiles for R.

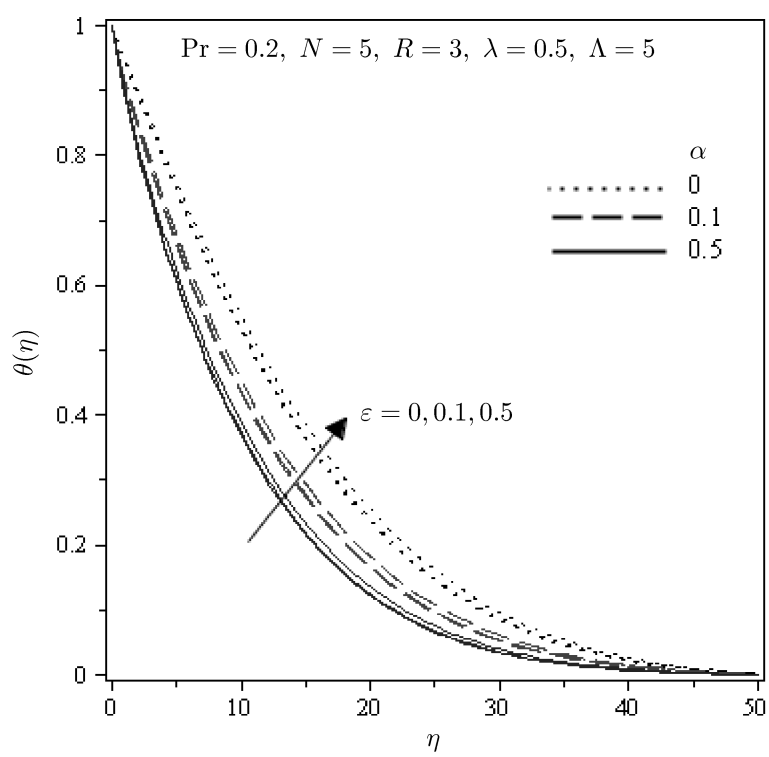

Fig. 4. Flow profiles ε.

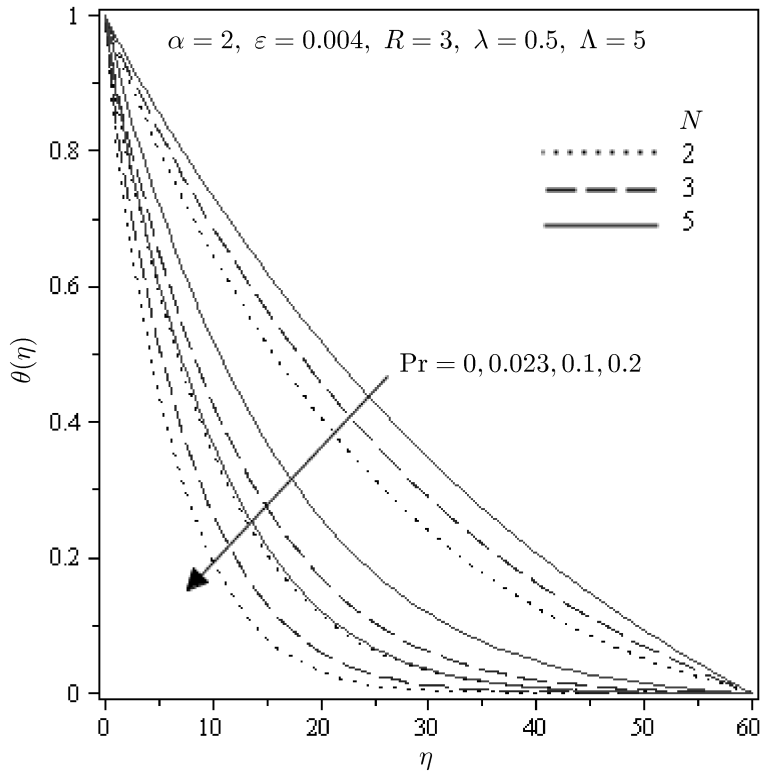

Fig. 5. Flow profiles for Pr.

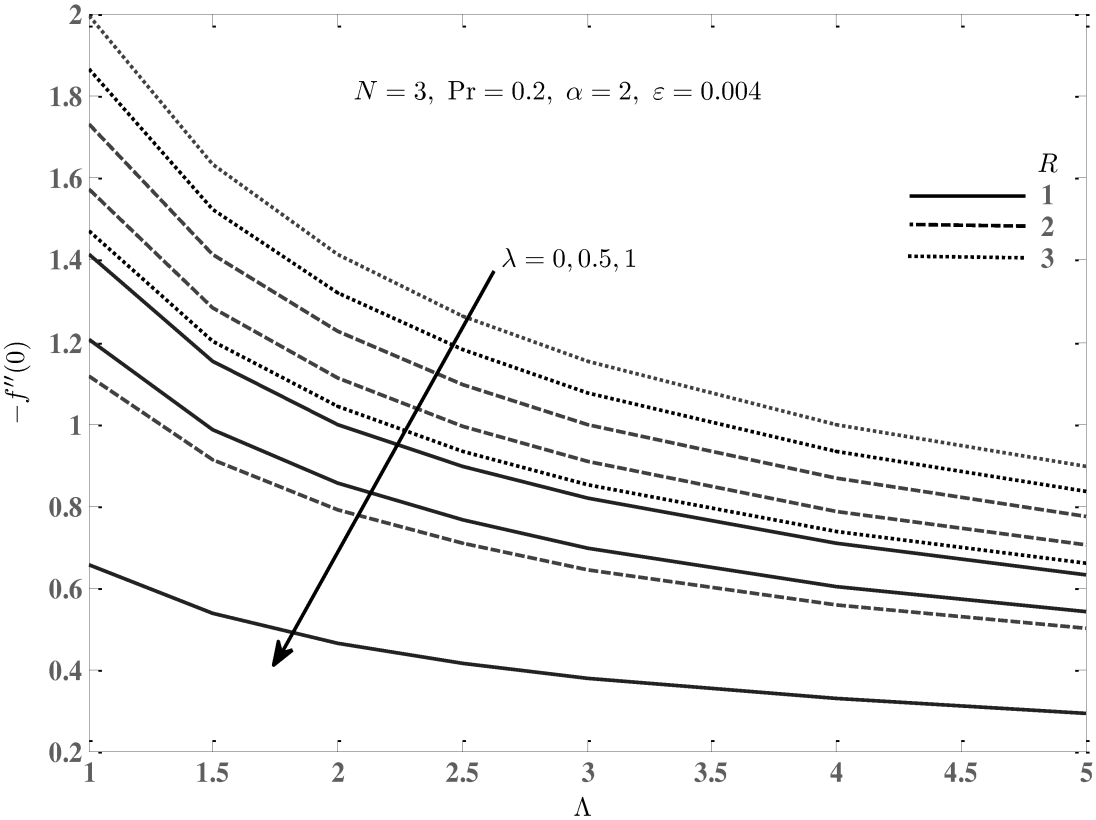

Fig. 6. Skin friction variation.

The variation of velocity and temperature distributions with the combined effect of magnetic field and porous diffusivity/permeability, i.e., R are shown in Fig. 3. The temperature boundary layers increase and the momentum boundary layers decrease as R increases. But for the velocity curves, we observe that near the stretching sheet, they decrease very rapidly, then remain constant at a certain value for each value of R and then start to increase and eventually agree with the boundary value.

In Fig. 4 we have plotted the dimensionless temperature profiles showing the effect of different values of wall temperature exponent α and perturbation parameter ε. It can easily be seen that the temperature profiles increase with the increase of ε but they decrease with the increase of α. This increase in the wall temperature exponent reduces the thermal boundary layer thickness and this conclusion was drawn by Tamayol et al. [24] in their study of thermal analysis of the flow in a porous medium over a permeable stretching surface. These authors obtained also an augmentation of the heat transfer when α increases.

Figure 5 expresses the effect of Prandtl number Pr and radiation parameter N on the temperature profiles. We observe that the thermal boundary layers decrease monotonically with the increase of Prandtl number Pr. However, the temperature profiles increase as radiation parameter N increases. It is the known physical fact that the boundary layer thickness decreases with increasing N, see, for instance, Mukhopadhyay [17].

Figure 6 shows the effect of Λ, λ, and R on the skin friction coefficient C_f, which is proportional to $f''(0)$. As Λ and λ increase, C_f increases. It is also observed that skin friction is reduced once a forced outer flow exists ($0 < \lambda \leq 1$). One learns consequently from this figure that not only the heat transfer can be controlled by flowing the fluid over the stretching sheet, but also the skin friction, leading to smaller resistive forces, which is important from the technological point of view.

On the other hand, as R increases, the skin friction coefficient decreases.

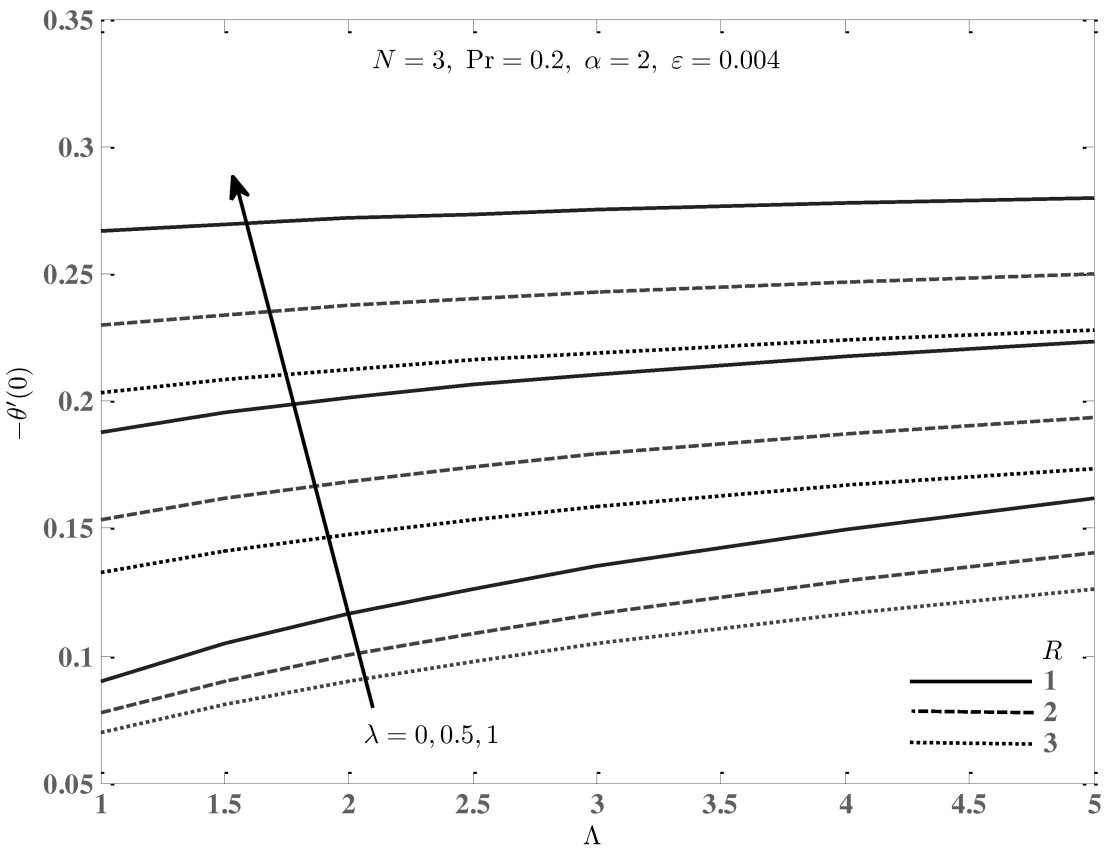

Fig. 7. Local Nusselt number variation.

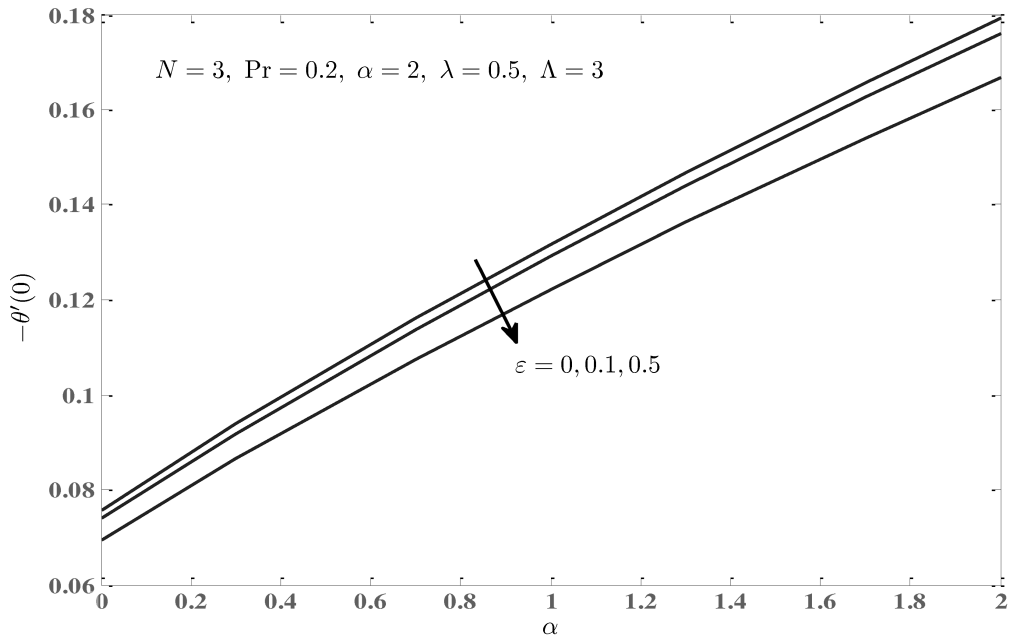

Fig. 8. Local Nusselt number variation.

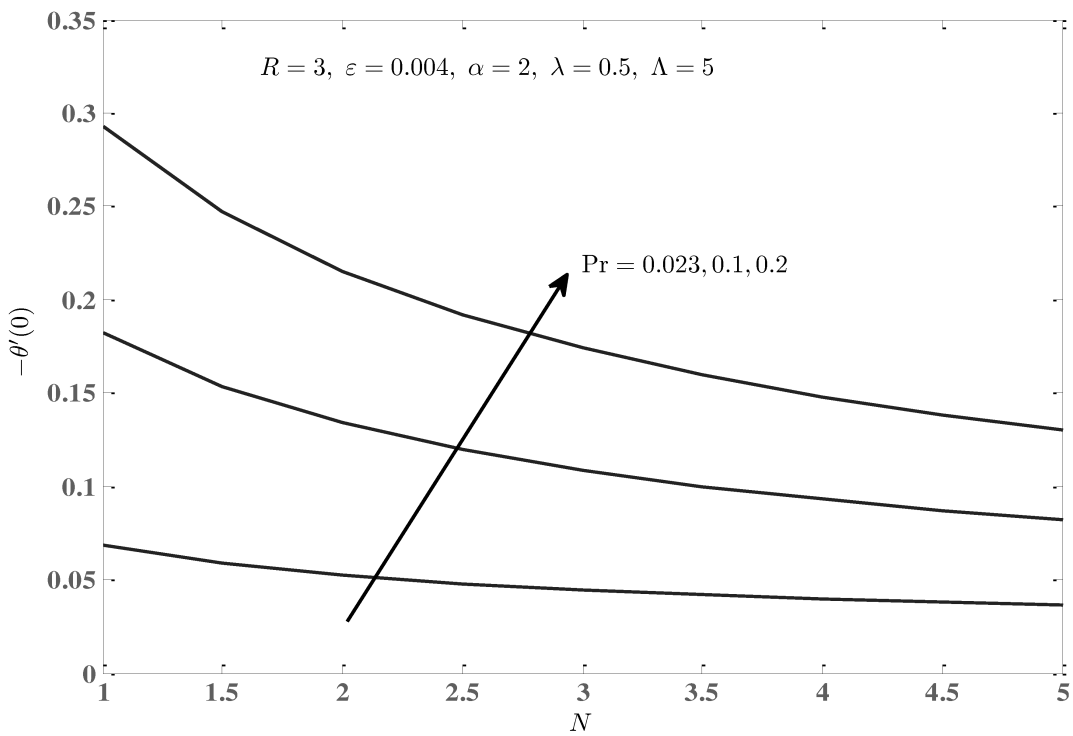

Fig. 9. Local Nusselt number variation.

Figure 7 exhibits the effect of Λ, λ, and R on the Nusselt number Nu, which is proportional to $-\theta'(0)$. As Λ increases, Nu increases slightly but it increases quite rapidly with the increase of λ. With regard to the first behavior, in comparison with Fig. 2, the influence of the viscosity ratio Λ is more significant on the Nusselt number, which tends to a constant value as Λ approaches higher values ($\Lambda = 5$ in this figure), and as the outer velocity equals, or exceeds the stretching velocity.

The effect of R observed in the same figure on the Nusselt number is to reduce the effect of transfer to the wall. Bearing in mind that this parameter embodies the permeability and the strength of the magnetic field, we have another proof of the possibility to control the heat transfer by means of an externally applied magnetic field.

Figure 8 displays the effect of α and ε on the Nusselt number Nu. Nusselt number Nu increases as α increases but Nu decreases with the increase of ε. Turning our attention to the effect of ε, illustrated in this figure and also in Fig. 4, it is worthwhile noticing that this parameter represents how much departs the thermal conductivity k from the reference value, which is in our k_∞. Indeed, from the definition of k, $k_\infty \leq k = k_\infty(1+\varepsilon)$. Definitely, our results show a reduction in the heat transfer to the wall (Fig. 8) and an intensification of the temperature fields (Fig. 4) as the difference between thermal conductivity at the wall and at the boundary layer edge becomes more substantial.

Figure 9 illustrates how Nusselt number Nu changes with Prandtl number Pr and radiation parameter N. Nusselt number Nu decreases with the increase of N but Nu increases with the increase of Pr.

4. CONCLUSIONS

(i) Velocity boundary layers increase with the increase of the ratio of the free stream velocity parameter to stretching sheet parameter λ and viscosity ratio Λ but decrease with the increase of the combined effect of porous diffusivity and magnetic field R.

(ii) Thermal boundary layers become thinner with the increase of the ratio of the free stream velocity parameter to stretching sheet parameter λ, viscosity ratio Λ, wall temperature parameter α and Prandtl number Pr (this latter behavior is wellknown).

(iii) The temperature profiles increase as the combined effect of porous diffusivity and magnetic field R, perturbation parameter ε and radiation parameter N increase.

(iv) Skin friction coefficient C_f increases with the increase of λ and Λ but decreases as R increases.

(v) Nusselt number Nu increases as λ, Λ, α, and Pr increase and behaves oppositely with the increase of R, ε, and N.

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